

TI questions 1-2:

1.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.
- (c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

2.

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

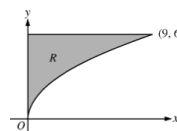
- (a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.
- (b) Find the x -coordinate of the particle's position at time $t = 4$.
- (c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.
- (d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

3.

No TI questions 3-6:

Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y -axis, as shown in the figure above.

- (a) Find the area of R .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
- (c) Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.



4. No TI questions 3-6:

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

- (a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.
- (b) At the point $(-1, 2)$, the value of $\frac{d^2y}{dx^2}$ is -12 . Find the second-degree Taylor polynomial for f about $x = -1$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

5.

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.

- (a) Estimate $f'(4)$. Show the work that leads to your answer.
- (b) Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
- (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.

6.

The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} + \dots$. The continuous function f is defined by

$f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function f has derivatives of all orders at $x = 1$.

- (a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.
- (b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.
- (c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).
- (d) Use the Taylor series for f about $x = 1$ to determine whether the graph of f has any points of inflection.