

TI questions 1-3:

1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.
- Find the area of R .
 - Find the volume of the solid generated when R is rotated about the x -axis.
 - The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.

2.

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

- The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \leq t \leq 7$.
- The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

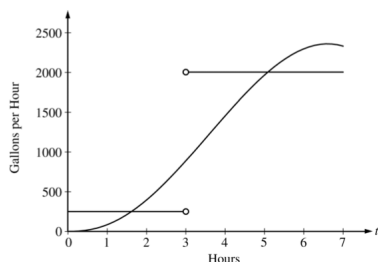


Fig 2 →

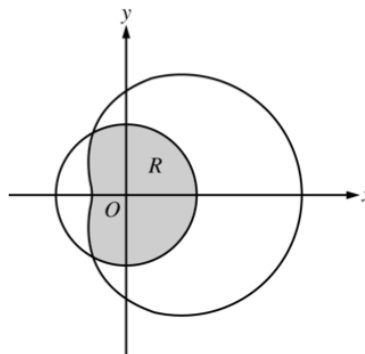


Fig 3 →

3.

The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

- Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R .
- A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t , with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.
- For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

No TI questions 4-6:

4.

Let f be the function defined for $x > 0$, with $f(e) = 2$ and f' , the first derivative of f , given by $f'(x) = x^2 \ln x$.

- Write an equation for the line tangent to the graph of f at the point $(e, 2)$.
- Is the graph of f concave up or concave down on the interval $1 < x < 3$? Give a reason for your answer.
- Use antidifferentiation to find $f(x)$.

5.

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$.

(Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

- Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
- Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
- Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
- Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.

6.

Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.

- Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.
- Is it possible for the x -axis to be tangent to the graph of f at some point? Explain why or why not.
- Find the second-degree Taylor polynomial for f about $x = -1$.
- Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.