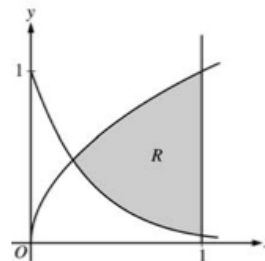


TI questions 1-3:

1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

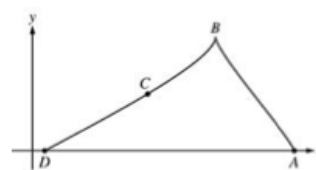
- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is 5 times the length of its base in region R . Find the volume of this solid.



2.

A particle starts at point A on the positive x -axis at time $t = 0$ and travels along the curve from A to B to C to D , as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t , where

$$x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right) \text{ and } y'(t) = \frac{dy}{dt} \text{ is not explicitly given.}$$



At time $t = 9$, the particle reaches its final position at point D on the positive x -axis.

- (a) At point C , is $\frac{dy}{dt}$ positive? At point C , is $\frac{dx}{dt}$ positive? Give a reason for each answer.
- (b) The slope of the curve is undefined at point B . At what time t is the particle at point B ?
- (c) The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.
- (d) How far apart are points A and D , the initial and final positions, respectively, of the particle?

3.

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table above shows the rate as measured every 3 hours for a 24-hour period.

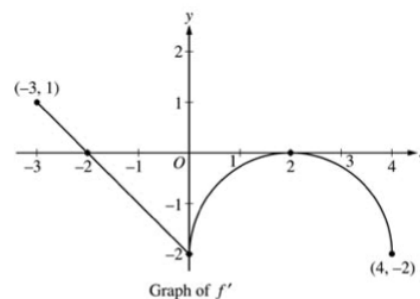
- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t) dt$. Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
- (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24 hour time period. Indicate units of measure.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

4. No TI questions 4-6:

Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.

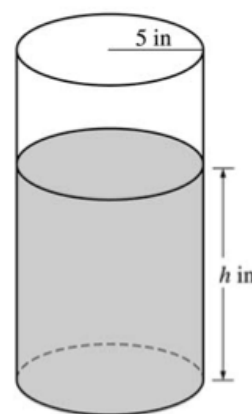
- On what intervals, if any, is f increasing? Justify your answer.
- Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
- Find an equation for the line tangent to the graph of f at the point $(0, 3)$.
- Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.



5.

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

- Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.
- Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .
- At what time t is the coffeepot empty?



6.

The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers x .

- Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.
- Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.
- Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.