1. Consider $f(x)$ to be a continuous, infinitely differentiable function. Let the family of Taylor polynomials for $f(x)$ centered at $x = a$ be called $p_k(x)$ where k is the order of the polynomial. We'll only consider values of x within the *radius of convergence*.

   (a) Write an equation for $p_o(x)$.

   (b) Explain what we mean when we say that each $p_k(x)$ has an associated remainder, $R_k(x)$, where $f(x) = p_k(x) + R_k(x)$. (Khan said this is also called E_k for *error*.)

   (c) What kind of object is $R_k(x)$?

   (d) Write an equation for $R_k(x)$ in terms of $f(x)$ and $p_k(x)$.

   (e) Write an equation for $R_o(x)$ in terms of $f(x)$ and $p_0(x)$.

   (f) Explain why $R_0(x) = \int_a^x f'(t)\,dt$.

2. Consider $f(x)$ to be a continuous, infinitely differentiable function, non-negative function. As you go through these questions, sketch pictures of graphs.

   (a) Does $f(x)$ ever change signs?

   (b) Which theorem says there must be some x_M on any closed interval $[a,b]$ where for all $x \in [a,b]$

   $$f(x) \leq f(x_M)$$

   (c) Now consider another function $g(x)$ and the integral $\int_a^b f(x)g(x)\,dx$. Explain why there must be some $x_M \in [a,b]$ such that

   $$\int_a^b f(x)g(x)\,dx \leq f(x_M)\int_a^b g(x)\,dx$$

   (d) Use part (c) to find an upper bound for $\int_0^1 \cos(x^2)\cdot x^2\,dx$. Explain.

3. Consider
$$I_2 = \int_a^x f^{(3)}(t)\frac{(x-t)^2}{2}\,dt$$
where $f(x)$ is a continuous, infinitely differentiable function, non-negative function. Use question $2(c)$ to find an upper bound. [You should be able to write the upper bound without an integral sign.]

4. Reconsider $I_2 = \displaystyle\int_a^x f^{(3)}(t)\frac{(x-t)^2}{2}\,dt$

   Evaluate I_2 using integration by parts where $u = f^{(3)}(t)$ and $dv = \dfrac{(x-t)^2}{2}\,dt$

5. Consider I_3

   (a) Write an equation for I_3 using a definite integral on the right side.

   (b) Evaluate I_3 using integration by parts in order to get I_4. (See question 4)

6. Repeat question 5 for I_4.