Looking for a Super Approximation

1. Our Data:

$$I = \int_1^3 x^{\frac{3}{2}}\,dx$$

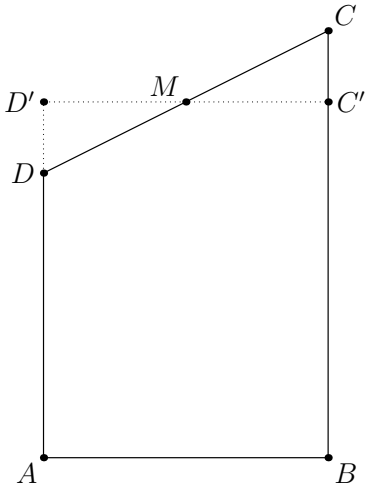
$$I \approx 5.835382907248$$

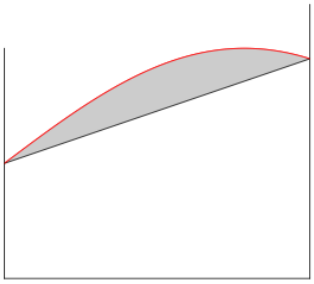
n	$M(n)$	$ME(n)$	$T(n)$	$TE(n)$	$S(h)$	$SE(n)$
4	5.82397	0.01142	5.85823	−0.02285		
8	5.83252	0.00286	5.84110	−0.00572		
16	5.83467	0.00071	5.83681	−0.00143		
32	5.83520	0.00018	5.83574	−0.00036		

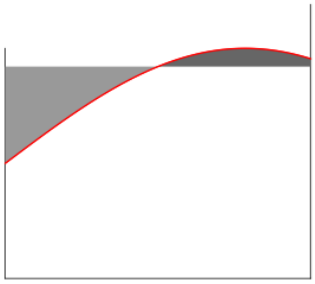
What can you say about the magnitudes and signs of the trapezoid and midpoint errors?

2. Some geometry

Consider trapezoid $ABCD$ and rectangle $ABC'D'$ where M is the midpoint of $\overline{DC}$. Explain why the trapezoid and rectangle have the same area.







3. Explain how these two pictures show TE_1 and ME_1.

4. Draw your own pictures to get an better way to compare TE_1 and ME_1 and confirm your answers to question 1.

5. Use the equations:

$$I = T + TE$$
$$I = M + ME$$

   to develop method that is our *Super Approximation* better than either the trapezoid or midpoint methods.

6. Go back to the table on the front, and use the Super result to fill in the missing values for $S(n)$ and $SE(n)$