Polar Area

1. Graph the constant polar function, $r(\theta) = R$ for $\theta \in [0, 2\pi)$.
   (a) What does a polar area element (dA) look like?
   (b) Use a proportion to write an equation for dA in terms of R and $d\theta$.

2. Consider $r(\theta) = 4\sin\theta$.
   (a) Setup a polar area element, dA.
   (b) Setup an integral and find the area.
   (c) Is your answer correct? (Again, no calculators)

Weighted Sums and Numeric Integrals

3. Professor Euler gives an exam to her two sections of linear algebra. The first section has an average score of 82.5 while the second section has an average score of 86.5.
   (a) What is the average score for all of her linear algebra students?
   (b) What if you know the first section has 20 students and the second section has 30 students?

4. Assume a university has 10 sections of Calculus I. Given the data below, find the average Calculus I test score.

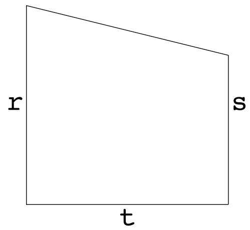
   Here are lists of the average scores and number of students for each section:

   scores = [81, 88, 82, 74, 78, 86, 83, 77, 82, 84.5]
   students = [31, 29, 37, 22, 34, 16, 25, 23, 29, 32]

   Note: The lists are ordered so the k^{th} entry in each list corresponds to section k.

5. Repeat the previous question for 139 sections when the average test score in section k is T_k and the number of students in section k is S_k.

6. Find the area of this right trapezoid. Explain why is your answer correct.



7. Consider $I = \int_1^3 x^{3/2}\,dx$

   (a) Use the FTC to determine an exact evaluation of this integral.

   (b) Complete the first and third rows of this table for left, right, and middle Riemann Sums as well as trapezoids that approximate our integral.

   (c) If A_n is an n-subinterval approximation for I, we can compute AE_n, the error for A_n, as $AE_n = I - A_n$. Use your answers to parts (a) and (b) to fill in the second and fourth rows showing the calculated errors in your approximations.

Boxes	Left Sum	Right Sum	Mid Sum	Trapezoids	
4	$L_4 =$	$R_4 =$	$M_4 =$	$T_4 =$	
4 error	$LE_4 =$	$RE_4 =$	$ME_4 =$	$TE_4 =$	
8					
8 error					

8. Show that T_4 for $\int_a^b f(x)\,dx$ can be calculated as

$$T_4 = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)) \text{ where } \Delta x = \frac{b-a}{4}.$$

9. Explain how T_n can be seen as a *weighted sum.*