1. Homer has to have an operation. When brought into the hospital, he is given a sedative to help him sleep. The doctor wants to operate, but cannot safely do so until the concentration of sedative in his body is less than 0.03 milligrams/liter. Your goal: how many hours the doctor must wait until she can operate?

   The following table of data was obtained by monitoring the levels of the sedative in Homer's blood. Samples were taken every ten minutes, and the concentration of the drug was determined and reported in milligrams per liter.

Time (*minutes*)	**Concentration** (*mg/L*)
0	10.000
10	6.070
20	3.680
30	2.230
40	1.350
50	0.820
60	0.498
70	0.302
80	0.183
90	0.111

   (a) Is the rate of change of concentration with respect to time a constant?

   (b) Estimate the rate of change of concentration with respect to time at 30 minutes, 60 minutes, and 80 minutes.

   (c) Show that the rate of change is (roughly) proportional to the concentration C. Write this relationship as a differential equation for $\frac{dC}{dt}$.

   (d) Find the constant of proportionality.

   (e) Solve the differential equation you wrote in part (c), using the constant of proportionality from part (d).

   (f) When is the drug's concentration halved?

   (g) If Homer received the sedative at 2:00 P.M., when can the doctor safely start the operation?

2. **Growing at Half Capacity**

   The *logistic growth model* is a model of population P versus time t given by the following differential equation: $\dfrac{dP}{dt} = kP\left(1 - \dfrac{P}{M}\right)$. In this exercise, we are concerned with the moment when the population P is equal to half the carrying capacity ($P = M/2$).

   (a) Without using your notes, demonstrate how to solve the differential equation.

   (b) Show that if the initial population is P_0, then $P = M/2$ when the time t is equal to

   $$T = \frac{1}{k} \ln \frac{M - P_0}{P_0}$$

   (c) What does $\dfrac{d^2P}{dt^2}$ represent in physical terms?

   (d) Show that the population is growing fastest at time T.