

1. Why is today called “ π -day” in some circles [sic]?

2. Read this conundrum:

Who, common in our words, from among a group of cardinality six fours and a two, is missing in this odd paragraph? Utilizing (by counting) this short and riddling wordplay that follows, you too can find which irrational constant is lacking in all this oratory.

*No radical, O constant of calculus;
a function of limiting form grows soaringly.*

At 18:28 this night, our shouts will soar as that function into air, for joy at this fantastic math constant.

What might have been the date (month/day) of publication for this passage?

3. Consider two polynomial functions, $f(x)$ and $g(x)$. If both polynomials have the same roots, can you be sure that the two functions are equivalent? If so, why? If not, what other information do you need? Be sure to discuss this with your group.

Our goal today is to evaluate $\sum_{k=1}^{\infty} \frac{1}{k^2}$. Don't get lost in the weeds.

4. *Pi notation* is to products as *sigma notation* is to sums. For example, just as

$$\sum_{k=1}^4 k = 1 + 2 + 3 + 4, \quad \prod_{k=1}^4 k = 1 \cdot 2 \cdot 3 \cdot 4. \text{ Of course that product can also be written as } 4!,$$

but not all products are factorials. Rewrite $\prod_{k=2}^5 (x+k)^{2k}$ without pi notation.

5. Factoring polynomials

(a) Show that the polynomial $\left(1 - \frac{x}{a}\right)\left(1 - \frac{x}{b}\right)$ has roots at $x = a$ and $x = b$. Show that the polynomial expands to $1 - \left(\frac{1}{a} + \frac{1}{b}\right)x + \frac{x^2}{ab}$.

(b) Show $a, -a, b,$ and $-b$ are roots of $\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{x^2}{b^2}\right)$ and that the expanded polynomial is $1 - \left(\frac{1}{a^2} + \frac{1}{b^2}\right)x^2 + \frac{x^4}{a^2b^2}$.

(c) Find the x^2 coefficient when $\left(1 - \frac{x^2}{a_1^2}\right)\left(1 - \frac{x^2}{a_2^2}\right)\left(1 - \frac{x^2}{a_3^2}\right)$ is expanded.

(d) Find the x^2 coefficient when $\prod_{k=1}^n \left(1 - \frac{x^2}{a_k^2}\right)$ is expanded. (Hint: Sigma notation is useful.)

6. Review our goal and your work on question 5. Now let's look for some useful functions and series.

(a) Use Π notation to write an equation for a function $f_n(x)$ that has zeroes at every positive and negative integer (zero is neither) from $-n$ to n .

(b) At the limit as n goes to infinity, $f_n(x)$ would have a single zero at each non-zero integer. Find a (well known) function that has zeroes at every integer (including zero).