Introducing Differential Equations

1. Equations
   (a) What is an equation?
   (b) What is a solution to an equation?
   (c) What type of object is a solution to an equations?
2. Differential Equations
   (a) What is a differental equation?
   (b) What is a solution to a differential equation?
   (c) What type of object is a solution to a differential equations?

3. A cup of black coffee is poured from a pot, whose contents are at 100 degrees (celsius), into a uninsulated cup in a room at 20 degrees. Let $y(t)$ be a function that gives the temperature of the coffee as a function of time.
   (a) Sketch a graph of $y(t)$. Place appropriate ticks on the y-axis, but not on the t-axis.
   (b) Write a differential equation which might describe the cooling behavior. [Hint: At any given point (t_a, y_a) on the graph of $y(t)$, what can you say about $y'(t_a)$?]

4. Solve these differential equations with initial conditions:
   (a) $f'(x) = 4x + 7$ with $f(3) = 19$.
   (b) $\dfrac{y'}{2\sin(t)} = \cos^3(t)$ with $y(0) = 7$.
   [y is a function of what variable?]

Old Questions

5. If $f(x)$ is an odd function then $\int_{-a}^{a} f(x)\,dx = 0$.
   (a) Draw a graph and explain why.
   (b) Use the algebraic definition of an odd function to explain why.
   Hint: Consider $G(x) = \int_{0}^{x} f(t)\,dt$. What do you need to show about $G(x)$?

6. Find the Taylor series for $f(x) = \ln(x)$ when $a = 1$.
   (a) Demonstrate how to do this by *brute force*, taking the derivatives of $\ln(x)$ at $x = 1$.
   (b) Demonstrate how to do this starting with the relationship of $f(x) = \dfrac{1}{1-x}$ and a simple converging geometric series.