

From Yesterday

1. Without consulting your notes, organize a table that shows

- the function
- the first 4 non-zero terms of the Maclaurin series
- the radius of convergence
- a column to be used later.

for these functions:

Function	Maclaurin series	radius of convergence	one more column
e^x			
$\sin(x)$			
$\cos(x)$			
$\frac{1}{1-x}$			
$\frac{1}{1+x}$			
$\ln(1+x)$			
$\frac{1}{1+x^2}$			
$\arctan(x)$			

2. These are converging Maclaurin series. For each series:

- Identify the function.
- Identify the value of x .
- Find the sum.

$$(a) 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

$$(b) 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$$

$$(c) \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$$

$$(d) 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

$$(e) 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots$$

$$(f) 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

3. For what values of x does this power series

$$\sum_{k=1}^{\infty} \frac{kx^k}{3^k}$$

converge. Explain your reasoning.

4. How many terms of the Maclaurin series for $\ln(1+x)$ are needed to estimate $\ln(1.5)$ to within 0.0001? [No calculators...]

5. Add a column to your table in question 1 to show sigma notation for each function. For example the series for e^x would be:

$$\sum_{k=0}^{\infty} \frac{x^k}{k!}$$

Note that *zero factorial* has a defined value of 1.

New Questions

6. Find the Taylor series for $f(x) = \ln(x)$ when $a = 1$.

(a) Demonstrate how to do this by *brute force*, taking the derivatives of $\ln(x)$ at $x = 1$.

(b) Demonstrate how to do this starting with the relationship of $f(x) = \frac{1}{1-x}$ and a simple converging geometric series.

7. If $f(x)$ is an odd function then $\int_{-a}^a f(x) dx = 0$.

(a) Draw a graph and explain why.

(b) Use the algebraic definition of an odd function to explain why.

Hint: Consider $G(x) = \int_0^x f(t) dt$. What do you need to show about $G(x)$?

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