

1. Demonstrate how to evaluate $L = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$.

2. Demonstrate how to build on the previous result (not method) to evaluate

$$K = \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^x.$$

3. If possible, we want to find the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$

- Consider the recursive sequence $a_0 = 0$, $a_{n+1} = \sqrt{1 + a_n}$. Compute the next five terms a_1 , a_2 , a_3 , a_4 , and a_5 . (You can use a calculator.)
- Were any of the values of a_k in the part (a) greater than 2?
- Explain how you can tell that $a_n < 2$ for all n . (If you understand mathematical induction ...)
- How do you know that $a_{n+1} > a_n$? (No formal proof is needed.)
- Since $\{a_n\}$ is increasing and bounded above by 2, the Monotone Sequence Theorem says that $\{a_n\}$ converges. If $\lim_{n \rightarrow \infty} a_n = a$, show that $a = \sqrt{1 + a}$.

(f) What is the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$?