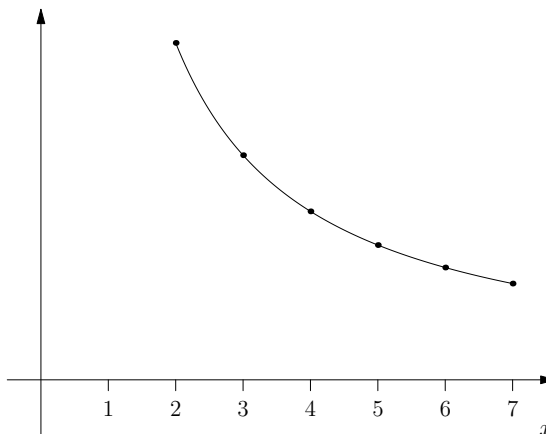


1. Use a graph white board to show:

$$\int_2^6 \frac{10}{x} dx > \sum_{k=3}^6 \frac{10}{k} > \int_3^7 \frac{10}{x} dx$$

Use colors and write an explanation.



2. Demonstrate two different ways to show whether  $\sum_{k=1}^{\infty} \frac{1}{k}$  converges or diverges.
3. A Fibonacci sequence is defined recursively as  $f_1 = 1, f_2 = 1, f_n = f_{n-1} + f_{n-2}$ .
- Write out the first 7 terms of the sequence.
  - Now consider the sequence  $\{r_k\}$  where  $r_k = \frac{f_{k+1}}{f_k}$ . Write out the first 7 terms of this sequence.
  - Should  $r_n$  converge as  $n \rightarrow \infty$ ? If so, find the limit. If not, explain how you know.

4. If possible, we want to find the value of  $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$

- Consider the recursive sequence  $a_0 = 0, a_{n+1} = \sqrt{1 + a_n}$ . Compute the next five terms  $a_1, a_2, a_3, a_4,$  and  $a_5$ . (You can use a calculator.)
- Were any of the values of  $a_k$  in the part (a) greater than 2?
- Explain how you can tell that  $a_n < 2$  for all  $n$ .
- How do you know that  $a_{n+1} > a_n$ ?
- Since  $\{a_n\}$  is increasing and bounded above by 2, the Monotone Sequence Theorem says that  $\{a_n\}$  converges. If  $\lim_{n \rightarrow \infty} a_n = a$ , show that  $a = \sqrt{1 + a}$ .

- What is the value of  $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$ ?