

1. **DoNow:** The Random Decimal

- (a) Have each person in your group think of a random integer from 0 through 9. Let a_1 be $0.wxyz$ where w, x, y and z are your numbers. For example, if you came up with 2, 4, 1, and 8, then you would write $a_1 = 0.2418$.
- (b) Have each person in your group think of a new integer, and add those integers to the end of a_1 to form a_2 . For example, if you already had $a_1 = 0.2418$, you might come up with $a_2 = 0.24185299$. Continue the process to form a_3, a_4 and a_5 .
- (c) If you continued this process infinitely many times, you would have an infinite sequence $\{a_n\}$. Does this sequence converge, diverge, or is it impossible to tell? Why?

2. A Fibonacci sequence is defined recursively as $f_1 = 1, f_2 = 1, f_n = f_{n-1} + f_{n-2}$.

- (a) Write out the first 7 terms of the sequence.
- (b) Now consider the sequence $\{r_k\}$ where $r_k = \frac{f_{k+1}}{f_k}$. Write out the first 7 terms of this sequence.
- (c) Should r_n converge as $n \rightarrow \infty$? If so, find the limit. If not, explain how you know.

3. If possible, we want to find the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$

- (a) Consider the recursive sequence $a_0 = 0, a_{n+1} = \sqrt{1 + a_n}$. Compute the next five terms a_1, a_2, a_3, a_4 , and a_5 . (You can use a calculator.)
- (b) Were any of the values of a_k in the part (a) greater than 2?
- (c) Explain how you can tell that $a_n < 2$ for all n .
- (d) How do you know that $a_{n+1} > a_n$?
- (e) Since $\{a_n\}$ is increasing and bounded above by 2, the Monotone Sequence Theorem says that $\{a_n\}$ converges. If $\lim_{n \rightarrow \infty} a_n = a$, show that $a = \sqrt{1 + a}$.

(f) What is the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$?