DoNow Two real numbers are chosen at random between 0 and 10. What is the probability that their sum is less than 5? is more than 10?

ReRevisiting Integrals

1. Assume $f(x)$ is a positive, non-linear, invertible function on the interval $[a, b]$ where $b > a > 0$ and $f(b) > f(a)$.

   (a) Sketch a possible graph of f on the domain $[a, b]$. Make sure your graph meets all of the criteria. On your graph, label a typical point (x, y) on the curve. Where is point $(x, f(x))$? Where is point $(f^{-1}(y), y)$? Explain.

   (b) Use your graph to show a geometric model of $A = \int_a^b f(x)\,dx$. Be sure to start with dA, a small piece of A, at your typical point (x, y) on your graph. Write a brief explanation.

   (c) Use your graph to show a geometric model of $B = \int_{f(a)}^{f(b)} f^{-1}(y)\,dy$. Start with dB, a small piece of B, at your typical point (x, y) on your graph. Write a brief explanation.

   (d) Use your graph to come up with a simple equation for the sum of the two integrals. [Hint: look for rectangles.]

   $$\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(y)\,dy = ??$$

   (e) Solve your equation [from part (d)] for $\int_a^b f(x)\,dx$.

   (f) Use your answer from part (e) to evaluate $\int_1^e \ln(x)\,dx$

2. Now for a non-geometric look:

   (a) Use integration by parts to show:

   $$\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$$

   (b) Use y-substitution to show:

   $$\int_a^b xf'(x)\,dx = \int_{f(a)}^{f(b)} f^{-1}(y)\,dy$$

   (c) Use parts (a) and (b) to show your result from question 1 part (e) is correct.

Homework Questions

3. Two real numbers, both between 0 and 2, are selected at random. What is the probability that their product is greater than 1?

4. What might $\int_0^\infty f(x)\,dx$ mean? Assuming $f(x) > 0$ for all x, could it (the integral) ever be finite?

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{5x}$.
   (a) Show that $y = \frac{1}{5}\ln(x)$ is a solution.
   (b) Show that $y = \frac{1}{5}\ln(5x)$ is a solution.
   (c) How can this be? Explain.

6. Evaluate:
   (a) $\int \sin^2(x)\,dx$
   (b) $\int \cos^3(t)\sin^4(t)\,dt$
   (c) $\int \cos^3(\theta)\,d\theta$
   (d) $\int \frac{5x+13}{x^2+5x+6}\,dx$
   (e) $\int \cos^6(2x)\sin^3(2x)\,dx$