

1. *Napkin Ring Problem – Revisited*: You discovered that the volume of a napkin ring is independent of radius of the original sphere. The volume is simply a function of h , the height of the napkin ring.

You explain this conclusion to our inquisitive eighth grader without telling her the formula for the napkin ring volume. She replies "that's cool! Let me see if I can use that fact to come up with a formula for $V(h)$, the volume of a napkin ring."

After giving the problem some thought, she comes back to tell you an equation for $V(h)$ and explains how she solved the problem without using calculus.

Demonstrate how she might have done this.

2. Evaluate this antiderivative $\int \sqrt{1-x^2} dx$ by using the θ -substitution $x = \sin(\theta)$.

(a) Test your answer on $\int_0^1 \sqrt{1-x^2} dx$.

(b) Evaluate $\int_0^{\frac{1}{2}} \sqrt{1-x^2} dx$.

3. Consider the graph of $f(x) = \sqrt{1-x^2}$. Confirm your understanding of using integrals to compute arc length (yes, *Particle, the Dog*) by finding the length of the graph of $f(x)$ for $x \in [0, 1]$. How can you check your answer?

4. *More on the Napkin Ring*

Consider slicing the napkin ring with height, h , and sphere radius, R , into washers. Setup a volume element, dV . Can you show that R does not matter by just considering the equation for dV without evaluating the integral? Explain.

5. Demonstrate how to evaluate $L = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$.

6. Demonstrate how to use the value of L from the previous question to evaluate

$$K = \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^x.$$

7. Use your work from another question $\left[\int \sqrt{1^2 - x^2} dx \right]$, to get this antiderivative,

$$\int \sqrt{a^2 - x^2} dx. \text{ Be sure to check your result.}$$