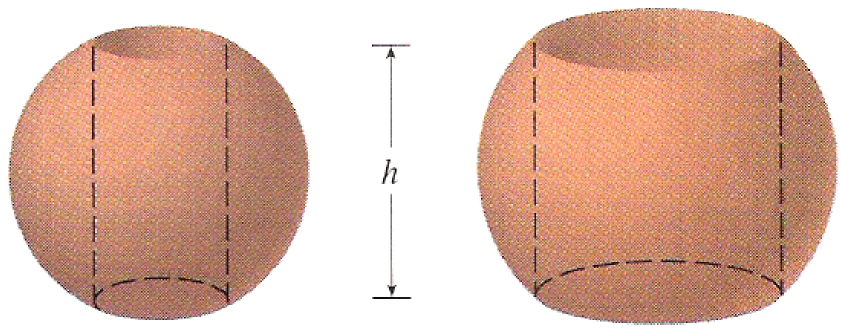
1. Napkin Ring Problem

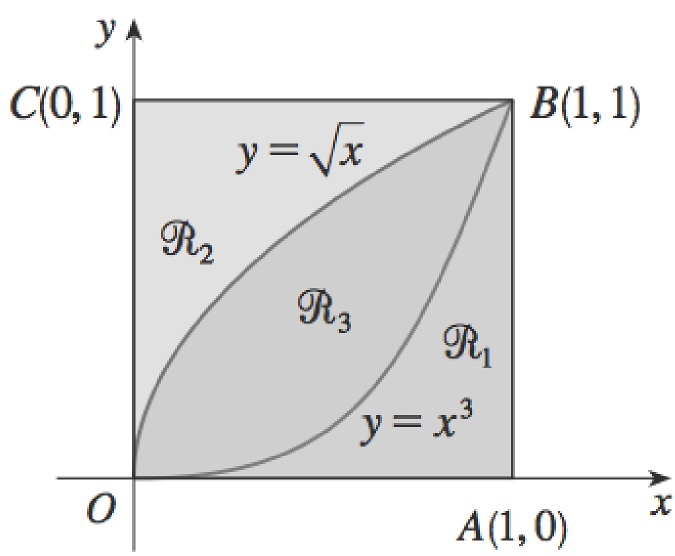


Use cylindrical shells to find a formula for the volume of a napkin ring created by drilling a hole through the center of a sphere of radius R leaving a height, h. Express your equation for V in terms of R and h.

2. Refer to the figure and consider the volume generated by rotating the given region about the specified line. For each subproblem below,

   (i) Use the diagram to setup a washer *volume element* (dV). Be sure to sketch an area element on region S and the graph of the given line.
   (ii) Set up (*but do not evaluate*) an integral to compute the volume of the solid obtained by rotating region S about the given line.

   (a) $\mathscr{R}_1$ about $\overline{OA}$
   (b) $\mathscr{R}_1$ about $\overline{AB}$
   (c) $\mathscr{R}_2$ about $\overline{OA}$
   (d) $\mathscr{R}_2$ about $\overline{AB}$
   (e) $\mathscr{R}_3$ about $\overline{OA}$
   (f) $\mathscr{R}_3$ about $\overline{AB}$



3. Repeat the previous question using cylindrical shells.

4. Demonstrate how to evaluate $\int \sin^2(\theta)\, d\theta$ using integration by parts.

5. *Napkin Ring Problem – Revisited:* You discovered that the volume of a napkin ring is independent of radius of the original sphere. The volume is simply a function of h, the height of the napkin ring.

   You explain this conclusion to our inquisitive eighth grader without telling her the formula for the napkin ring volume. She replies "that's cool! Let me see if I can use that fact to come up with a formula for $V(h)$, the volume of a napkin ring."

   After giving the problem some thought, she comes back to tell you an equation for $V(h)$ and explains how she solved the problem without using calculus.

   Demonstrate how she might have done this.

6. **Particle, the Dog:**
   My dog, Particle, and I like to walk on the Cartesian plan. Like most dogs, Particle walks further than I do, even though we start and end in the same places. Unlike most dogs, Particle walks along a path described by a sine curve.

   As I walk along the x-axis, Particle moves along the curve $y = \sin x$, so that her x is the same as mine. For now, your goal is to develop distance function, $P(x)$, that takes a value of x and gives the "distance" Particle has walked along the sine curve from her starting point at the origin.

   (a) Develop a *distance element*, dP, for computing Particle's distance. Use this to come up with a method to obtain how far Particle has walked from the origin when I am at any point x. [Note that you will not need to actually give a numerical answer to this part.]