DoNow Given a cone with height H and base radius R, setup a volume element dV, an integral, and evaluate the integral to show that the volume of the cone is $1/3\pi R^2 H$.

Revisiting Limits and Indeterminate Expressions

The goal here is to understand a different way to determine limits of an expression whose "value" would be $0/0$ by a simple substitution. Recall we saw this when we looked at $\lim_{x\to 0}\frac{\sin x}{x}$. In that case we used the *squeeze theorem*.
This time consider

$$\lim_{x\to 1}\frac{\ln x+\sin(x-1)}{1-e^{x-1}}$$

1. Prepare your calculator with

$$\begin{aligned} g(x) &= \ln x+\sin(x-1) \\ h(x) &= 1-e^{x-1} \\ f(x) &= \frac{g(x)}{h(x)} \end{aligned}$$

   Zoom decimal on the TI is a reasonable window. Be sure to record the correspondence between the functions g, h and f and the calculator Y_i names.

2. What is the value of $f(1)$?
   What is a reasonable guess for the limit of $f(x)$ as x goes to 1?

3. What is the equation for the line tangent to $g(x)$ at $g(1)$?
   Add the tangent line to the screen of your calculator. Call this $T_g(x)$.
   (Why is $T_g(x)$ a good name?)

4. What is the equation for the line tangent to $h(x)$ at $h(1)$?
   Add the tangent line to the screen of your calculator. Call this $T_h(x)$

5. Turn off $f(x)$ and zoom in on $(1,0)$.
   What is happening with $T_g(x)$ (the tangent at $g(1)$) and $g(x)$?
   What about $T_h(x)$ and $h(x)$?
   How close can you make $T_g(x)$ to $g(x)$ in the neighborhood of $x=1$?

6. Consider using $\lim\limits_{x\to 1}\dfrac{T_g(x)}{T_h(x)}$ as a replacement for $\lim\limits_{x\to 1}\dfrac{g(x)}{h(x)}$.
   Is it any easier to evaluate $\lim\limits_{x\to 1}\dfrac{T_g(x)}{T_h(x)}$?
   Can you see a common factor in both $T_g(x)$ and $T_h(x)$?
   How might this help you?

7. Use this method on $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$. Before you begin, try to generalize the steps you took above to transform an indeterminate limit into one that could be determined. *E.g.,* how did you construct $T_g(x)$ and $T_h(x)$?
   What is left once you did the factoring in $\dfrac{T_g(x)}{T_h(x)}$?

8. **L'Hôpital's Rule** is a method for evaluating limits of an indeterminate form of type $\frac{0}{0}$. Describe your concept of L'Hôpital's Rule.

Using L'Hôpital's Rule

9. Here is an example of notation for L'Hôpital's Rule.

$$\lim_{x\to 0}\frac{\sin(x)}{x} \overset{H}{=} \lim_{x\to 0}\frac{\cos(x)}{1} = \frac{1}{1} = 1$$

   Demonstrate how to use L'Hôpital's Rule to evaluate:

   (a) $\lim\limits_{x\to 0}\dfrac{\cos^2 x - 1}{2x^2}$

   (b) $\lim\limits_{x\to\infty}\dfrac{3x^4 - 12x^2 + 7}{15x^4 + 17x^3 - 3x^2 + 17}$

   (c) $\lim\limits_{x\to 0} x\cdot\ln(x)$

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