Revisiting Derivatives of Area Functions

1. Let $F(x) = \int_0^x f(t)\,dt$, $G(x) = \int_1^x f(t)\,dt$, and $H(x) = \int_{-2}^x f(t)\,dt$.
   Suppose $f(x) = 3x$.
   (a) Sketch a graph of $f(x)$.
   (b) Find equations for $F(x)$, $G(x)$, and $H(x)$ that do not use an integral sign.
   (c) Find $F'(x)$, $G'(x)$, and $H'(x)$. Notice anything?

2. Let $F(x) = \int_a^x f(t)\,dt$ and $G(x) = \int_b^x f(t)\,dt$, where a and b are constants, and f is a continuous function.

   Use properties of definite integrals to show that $G(x) = F(x) + C$ where C is a constant.

3. Make a conjucture:

   If $A(x) = \int_a^x f(t)\,dt$ then $A'(x) =$ __________

   Use your understanding of *area functions* and derivatives to explain why your conjecture is true.

4. Discuss with your group how you might prove your conjecture.

Don't start this side until we as a class have gone over the first side

5. Evaluate $A'(x)$ when:
   (a) $A(x) = \int_0^x t^2\,dt$ (b) $A(x) = \int_3^x t^3\,dt$

   (c) $A(x) = \int_x^5 \ln(t)\,dt$ (d) $A(x) = \int_3^{x^2} 7^t\,dt$

Related Rates Problems (questions numbered to match homework)

1. The radius of a circular puddle is 3 *meters* and it is increasing at the rate of 1 *cm/min*. How fast is the puddle's area increasing?

2. A bag is tied to the top of a 5 $meter$ long ladder which is resting against a vertical wall. Suppose the ladder begins sliding down the wall in such a way that the foot of the ladder is moving away from the wall. How fast is the bag descending at the instant the foot of the ladder is 4 $meters$ from the wall and the foot of the ladder is moving away from the wall at the rate of 2 $meters/sec$?

3. A 6 ft tall person is walking away from a streetlight that is 20 ft high, at the rate of 7 ft/sec. At what rate is the person's shadow increasing?

4. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4pm.

5. A tank filled with water is in the shape of an inverted cone 20 ft high with a circular base, on top, whose radius is 5 ft. Water is running out of the bottom of the tank at a constant rate of $2ft^3/min$. How fast is the water level falling when the water is 8 ft deep?

6. A person is standing on the ground watching a jet through a telescope as it approaches at a speed of 10 $miles/minute$ at an altitude of 7 $miles$. At what rate (in $radians/minute$) is the angle of the telescope changing when the horizontal distance of the jet from the woman is 24 $miles$? When the jet is directly above the person?