From Yesterday (with additional part iii questions)

1. Let $F(x) = \int_0^x f(t)\,dt$, $G(x) = \int_1^x f(t)\,dt$, and $H(x) = \int_{-2}^x f(t)\,dt$.

   (a) Suppose $f(x) = 1$.

   i. Sketch a graph of $f(x)$.

   ii. Find equations for $F(x)$, $G(x)$, and $H(x)$ that do not use an integral sign.

   iii. Find $F'(x)$, $G'(x)$, and $H'(x)$. Notice anything?

   (b) Suppose $f(x) = 3x$.

   i. Sketch a graph of $f(x)$.

   ii. Find equations for $F(x)$, $G(x)$, and $H(x)$ that do not use an integral sign.

   iii. Find $F'(x)$, $G'(x)$, and $H'(x)$. Notice anything?

   (c) Suppose $f(x) = 3x + 1$.

   i. Sketch a graph of $f(x)$.

   ii. Find equations for $F(x)$, $G(x)$, and $H(x)$ that do not use an integral sign. [Hint: Use parts (a) and (b).]

   iii. Find $F'(x)$, $G'(x)$, and $H'(x)$. Notice anything?

2. Let $F(x) = \int_a^x f(t)\,dt$ and $G(x) = \int_b^x f(t)\,dt$, where a and b are constants, and f is a continuous function.

   Use properties of definite integrals to show that $G(x) = F(x) + C$ where C is a constant.

3. **Both Zeros?**

   (a) Assume $x + y = 0$. Can we say the both x and y must be zero?

   (b) Assume $s^2 + t^2 = 0$. Can we say the both s and t must be zero?

4. **A pair functions and their derivatives**

   Suppose that f and g are functions such that:

   (i) $f(0) = 0$ (ii) $g(0) = 1$
   (iii) $f'(x) = g(x), \forall x$ (iv) $g'(x) = -f(x), \forall x$
   (where $\forall$ is a symbol for *for all*.)

   (a) You might recognize that $\sin(x)$ and $\cos(x)$ are a pair of functions that meet the descriptions of $f(x)$ and $g(x)$. Check that this is true.

   (b) We know that $\sin^2(x) + \cos^2(x) \equiv 1$. Perhaps this is true for all functions f and g which meet the description above.
   Let $h(x) = (f(x))^2 + (g(x))^2$. Show $h(x) = 1, \forall x$. [Hint: derivatives should help.]

   (c) Are there really non-trig functions that work? Assume f and g are another pair of functions that satisfy conditions (i)-(iv).
   Let $k(x) = (\sin(x) - f(x))^2 + (\cos(x) - g(x))^2$. What can you say about $k(x)$?