DoNow

1. Without a calculator give a good (accurate to three decimal places) approximation for $\sin(.003)$.
   (a) Why do you think your approximation is good?
   (b) Without looking at a better a better approximation (e.g. a calculator value), is your approximation *greater than* or *less than* the actual value? Explain why your are correct.

2. What is the *average value* of the numbers in this list:
   $$1, 2, 2, 3$$
   Make a sketch using dots to show a *geometric interpretation* of your answer.

3. Find the average value of $\cos(x)$ on the closed interval $[0, \pi]$. Explain.

4. Average Value of another Step-Function
   $$g(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1, \\ 2 & \text{if } 1 < x \le (2+\sqrt{2}), \\ 3 & \text{if } (2+\sqrt{2}) < x \le 6. \end{cases}$$
   (a) Draw a graph of g for $x \in [0, 6]$.
   (b) What is the average value of $g(x)$ for $x \in [0, 6]$.
   (c) Color your graph to demonstrate a *geometric* visualization of the *average value* of g over the interval $[0, 6]$.

Area Functions

Recall that *area* is a geometric concept. Think geometry!

5. Let $F(x) = \int_0^x f(t)\,dt$, $G(x) = \int_1^x f(t)\,dt$, and $H(x) = \int_{-2}^x f(t)\,dt$.

   (a) Suppose $f(x) = 1$.

      i. Sketch a graph of $f(x)$.

      ii. Find equations for $F(x)$, $G(x)$, and $H(x)$ that do not use an integral sign.

   (b) Suppose $f(x) = 3x$.

      i. Sketch a graph of $f(x)$.

      ii. Find equations for $F(x)$, $G(x)$, and $H(x)$ that do not use an integral sign.

   (c) Suppose $f(x) = 3x + 1$.

      i. Sketch a graph of $f(x)$.

      ii. Find equations for $F(x)$, $G(x)$, and $H(x)$ that do not use an integral sign. [Hint: Use parts (a) and (b).]

6. Let $F(x) = \int_a^x f(t)\,dt$ and $G(x) = \int_b^x f(t)\,dt$, where a and b are constants, and f is a continuous function.

   Use properties of definite integrals to show that $G(x) = F(x) + C$ where C is a constant.