Questions From Yesterday to Present

1. Use diagrams and our geometric interpretation of a definite integral to explain why:

   If $g(x) \geq f(x)$ for all $x \in [a,b]$ then $\displaystyle\int_a^b g(x)\,dx \geq \int_a^b f(x)\,dx$

2. Use the properties of definite integrals to show:

   If $m \leq f(x) \leq M$ for all $x \in [a,b]$ then

   $$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

3. Sketch the graph of a function f with the property that:

   $$\left| \int_1^5 f(x)\,dx \right| < \int_1^5 |f(x)|\,dx$$

4. Show that $\displaystyle\int_0^{\pi/2} \sqrt{1+\cos(2x)}\,dx = \sqrt{2}\int_0^{\pi/2} \cos x\,dx$

5. Show that $\dfrac{\pi}{2} < \displaystyle\int_0^{\pi} \cos(\sin x)\,dx \leq \pi$
   (Without using a calculator.)

6. Consider some function $f(x)$ where $f''(x)$ is continuous on the closed interval $[2,5]$, $f''(3) = 0$, and $f''(x) \neq 0$ for $x \in [2,3)$ and $x \in (3,5]$.

   (a) Sketch some possible graphs for $f''(x)$.

   (b) Is it possible that $(3, f(3))$ is not an *inflection point* for $f(x)$? If so, give an example by drawing a picture. If not, explain why.

7. Given $f'(x) = \cos(x)$,

   (a) Find a possible equation for $f(x)$.

   (b) Are there any other correct answers part (a)? Explain.

Things to work on

8. **Going from rate functions to amount functions**

   (a) Given $f'(x) = 3x^2$, find possible equation(s) for $f(x)$.

   (b) Given $f'(x) = x^2$, find possible equation(s) for $f(x)$.

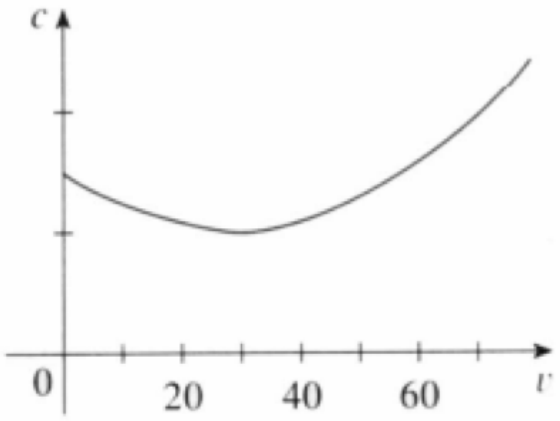
   (c) Given $f'(x) = \sin(x)$, find possible equation(s) for $f(x)$.

   (d) Given $f'(x) = \sin(2x)$, find possible equation(s) for $f(x)$.

   (e) Given $f'(x) = \frac{1}{x}$, find possible equation(s) for $f(x)$.

9. **Fuel Consumption Problem**

   The graph below shows the fuel consumption c of a car (measured in gallons per hour) as a function of the speed v of the car. At very low speeds the engine runs inefficiently, so initially c decreases as the speed increases. But at high speeds the fuel consumption increases. You can see that $c(v)$ is minimized for this car when v is about 30 mi/h. However, for fuel efficiency, what must be minimized is not the consumption in gallons per hour but rather the fuel consumption in gallons per mile. Let's call this consumption G. Using the graph, estimate the speed at which G has its minimum value.



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