Properties of Definite Integrals

1. $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
2. $\int_a^b k\cdot f(x)\,dx = k\int_a^b f(x)\,dx$
3. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
4. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$
5. If $g(x) \geq f(x)$ for all $x \in [a,b]$ then $\int_a^b g(x)\,dx \geq \int_a^b f(x)\,dx$

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1. Draw a picture for each of the following definite integrals.
   (a) $\int_1^3 4\,dx$ (b) $\int_0^2 x\,dx$ (c) $\int_1^1 \theta e^{\theta}\,d\theta$
   (d) $\int_1^3 -4\,dx$ (e) $\int_a^b 4\,dx$ (f) $\int_a^b c\,dx$
2. Evaluate (i.e. give *exact* numbers, not approximations) each of the definite integrals in problem 1.
3. Explain why: $\int_1^2 x^3\,dx = \int_3^4 (x-2)^3\,dx = \int_{-3}^{-2} (x+4)^3\,dx$
4. Assume $\int_0^2 g(x)\,dx = 2$; $\int_1^2 g(x)\,dx = -1$; and $\int_2^4 g(x)\,dx = 7$. Evaluate:
   (a) $\int_1^4 g(x)\,dx$ (b) $\int_0^4 3g(x)\,dx$ (c) $\int_2^1 g(x)\,dx$
   (d) $\int_0^1 g(x)\,dx$ (e) $\int_0^1 g(x+1)\,dx$ (f) $\int_0^1 \big(g(x)+1\big)\,dx$
   (g) $\int_2^4 g(x-2)\,dx$ (h) $\int_2^4 \big(g(x)-2\big)\,dx$