DoNow

1. A line with positive slope makes an angle with the x-axis. What is the relationship between the slope of the line and the angle? [Draw a picture.]

Derivatives of Trig Functions

2. Given $\lim_{x\to 0}\dfrac{\sin x}{x} = 1$, show $\lim_{x\to 0}\dfrac{1-\cos x}{x} = 0$.

   [Hint: look for a clever value of 1 to use as a multiplier.]

3. (a) Use a grid white board to make a reasonable sketch $\sin(x)$ over $[0, 2\pi]$ by:
      i. Use a straight edge to draw the horizontal line segment that bisects the grid and use this as the x-axis. Label the left end of that line segment as 0 and the right end as 2π.
      ii. Draw (with a straight edge) the y-axis. Divide the y-axis into quarters and label positions -2, -1, 1, and 2 to the left of the y-axis.
      iii. Plot points for $\sin(x)$ at 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π, $\frac{5\pi}{4}$, $\frac{3\pi}{2}$, $\frac{7\pi}{4}$, 2π.
      iv. Make the points large enough to be seen even after you sketch the graph of $\sin x$
   (b) Sketch the derivative function over the same interval.
   (c) Take a guess on the derivative function.
   (d) Use the difference quotient (and some well known limits) to find an expression for $\dfrac{d(\sin(x))}{dx}$.

4. Repeat the process to get a guess and an expression for $\big(\cos(x)\big)'$.

5. Now use your results to find $\big(\tan(x)\big)'$.

   [Hint: You don't need to use the difference quotient. How can you rewrite $\tan(x)$ in terms of $\sin(x)$ and $\cos(x)$?]

Practice: Assume that $f(x)$ and $g(x)$ are differentiable functions about which we know very little. In fact, assume that all we know about these functions is the following table of data:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	?

This isn't a lot of information. For example, we can't compute $f'(3)$ with any degree of accuracy. But we are still able to figure some things out, using the rules of differentiation.

1. Let $h(x) = e^x f(x)$. What is $h'(0)$?
2. Let $j(x) = -4f(x)g(x)$. What is $j'(1)$?
3. Let $k(x) = \dfrac{xf(x)}{g(x)}$. What is $k'(-2)$?
4. Let $l(x) = x^3 g(x)$. If $l'(2) = -48$, what is $g'(2)$?
5. Let $n(x) = x^2 f(x)g(x)$. What is $n'(1)$?

IVT

5. The *intermediate value theorem* states:

   > If f is a real-valued continuous function on the interval $[a,b]$, and u is a number between $f(a)$ and $f(b)$, then there is a $c \in [a,b]$ such that $f(c) = u$.

   Right now (and until you take a math course called *Real Analysis*), take the IVT as a postulate.

   (a) Explain what the IVT means. Be sure to draw at least one picture.

   (b) Recall there is no *rational number* whose square is 2. Explain how the IVT shows that there is a *real* number whose square is 2.

6. A hiker sets out at dawn to walk over a mountain and down the other side to a lake, a distance of x kilometers, and she reaches her destination exactly 12 hours later. She camps, then leaves at the same time the next morning and travels the same trail back, arriving at her starting place exactly 12 hours later. Use the *Intermediate Value Theorem* to show that at some specific time, she was at the same place on her hike on both days.