

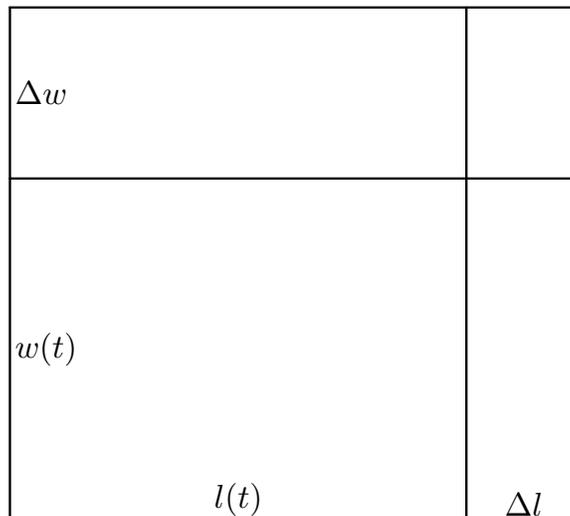
1.  $f(x) = ax^2 + bx + c$ 
  - (a) In general terms, describe the graph of  $f(x)$ .
  - (b) Find  $f'(x)$ .
  - (c) Find the stationary point(s) of  $f(x)$ .
  - (d) Is there a special name for a stationary point of this particular function?

2. Products

- (a) Let  $f(x) = x^5$ . Find  $f'(x)$
- (b) Let  $u(x) = x^3$ . Find  $u'(x)$
- (c) Let  $v(x) = x^2$ . Find  $v'(x)$
- (d) Is the derivative of a product the same as the product of the derivatives?

3. Consider a rectangle where the width is given by  $w(t)$  and the length is  $l(t)$ . The area, at any time,  $t$ , is the function  $A(t) = l(t) \cdot w(t)$ . Let  $\Delta A = A(t + \Delta t) - A(t)$ . Use this diagram to find  $\frac{\Delta A}{\Delta t}$  in terms of  $l(t), \Delta l, w(t), \Delta w$  and  $\Delta t$ .

At time  $t$ , our rectangle is just the one in the lower left. At time  $t + \Delta t$ , it has grown to be the largest rectangle.



(a) Find the area of the largest rectangle and each of the four small rectangles.

(b) Use part (a) to write an equation for  $\frac{\Delta A}{\Delta t}$

(c) Now express  $\frac{dA}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta A}{\Delta t}$  in terms of the functions  $l(t), l'(t), w(t)$  and  $w'(t)$ .

4. Use a difference quotient to find  $g'(a)$  when:

(i)  $g(x) = \sqrt{x}$

(ii)  $g(x) = \frac{1}{x^2}$

5. The *intermediate value theorem* states:

If  $f$  is a real-valued continuous function on the interval  $[a, b]$ , and  $u$  is a number between  $f(a)$  and  $f(b)$ , then there is a  $c \in [a, b]$  such that  $f(c) = u$ .

Right now (and until you take a math course called *Real Analysis*), take the IVT as a postulate.

- (a) Explain what the IVT means. Be sure to draw at least one picture.
- (b) Recall there is no *rational number* whose square is 2. Explain how the IVT shows that there is a *real* number whose square is 2.

6. A hiker sets out at dawn to walk over a mountain and down the other side to a lake, a distance of  $x$  kilometers, and she reaches her destination exactly 12 hours later. She camps, then leaves at the same time the next morning and travels the same trail back, arriving at her starting place exactly 12 hours later. Use the *Intermediate Value Theorem* to show that at some specific time, she was at the same place on her hike on both days.