

**Sneaking Up on**  $\frac{\sin 0}{0}$

1. Use a unit circle to observe that  $\sin(x)$  is an *odd function*.
2. Use our limit from the positive side  $\lim_{x \rightarrow 0^+} \frac{\sin x}{x}$  and the symmetry of  $\sin(x)$  to show:

$$\lim_{x \rightarrow 0^-} \frac{\sin x}{x}$$

Hint: Start with  $\lim_{u \rightarrow 0^+} \frac{\sin u}{u}$  and let  $u = -x$ .

3. Given  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ , demonstrate how to use a difference quotient to find  $\sin'(0)$ .
4. Are there are values of  $x$  where  $|x| < |\sin(x)|$ ? Explain.

**Using the IVT**

5. The *intermediate value theorem* states:

If  $f$  is a real-valued continuous function on the interval  $[a, b]$ , and  $u$  is a number between  $f(a)$  and  $f(b)$ , then there is a  $c \in [a, b]$  such that  $f(c) = u$ .

Right now, take the IVT as a postulate.

- (a) Explain what the IVT means. Be sure to draw at least one picture.
  - (b) Recall there is no *rational number* whose square is 2. Explain how the IVT shows that there is a *real number* whose square is 2.
6. A hiker sets out at dawn to walk over a mountain and down the other side to a lake, a distance of  $x$  kilometers, and she reaches her destination exactly 12 hours later. She camps, then leaves at the same time the next morning and travels the same trail back, arriving at her starting place exactly 12 hours later. Use the *Intermediate Value Theorem* to show that at some specific time, she was at the same place on her hike on both days.