Logs

1. What are some relationships between the functions $f(x) = \log_2(x)$ and $g(x) = 2^x$? Explain.
2. The *kilo* in "kilobyte" is 1,024 rather than the usual 1,000 as in "kilometer."
   (a) Without a calculator, show 1,024 to be an integer power of 2.
   (b) Show how to use part (a) to get a good estimate of how many dollars are equivalent to 2^{30} cents.
   (c) Show how to use part (a) to estimate the power to which 10 must be raised to yield 2.
3. Using definitions
   (a) Write a definition for a logarithm. [You can start with the equation $q = \log_r(s)$.]
   (b) Use this definition of a log to show $\log_b(a^2) \equiv 2\log_b(a)$.

From Yesterday

4. Functions with symmetries:
   (a) Give a geometric definition of an *even* function. Sketch an example.
   (b) Give a geometric definition of an *odd* function. Sketch an example.
5. Algebraically, we define $E(x)$ to be an *even* function if $E(-x) = E(x)$ for all values of x in the domain of E. Using this example, give an algebraic definition of an *odd* function, $O(x)$.
6. Suppose $E(x)$ is an even function, $O(x)$ is an odd function, and $f(x)$ is any function. Explore these composition functions and determine if they *always* have *even* symmetry.
   • If you think the answer is *yes*, use the definition to prove your conjecture.
   • If you think the answer is *no*, show a counter-example.
   (a) $E \circ O(x)$
   (b) $E \circ f(x)$

7. Recall the limit rules (e.g. *the limit of a product is the product of the limits*). In English, write the proposed derivative rule, and use a difference quotient [see your bathroom mirror] to show:

   (a) If k is a constant and $g(x) = kf(x)$ then $g'(x) = kf'(x)$.

   (b) If $s(x) = f(x) + g(x)$ then $s'(x) = f'(x) + g'(x)$