1. Use integration by parts to show:

$$\int \sin^4(x)\,dx = \frac{\sin^3(x)\cos(x)}{4} + \frac{3}{4}\int \sin^2(x)\,dx.$$

2. We can generalize the result into this *reduction formula*:

$$\int \sin^n(x)\,dx = \frac{\sin^{n-1}(x)\cos(x)}{n} + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx.$$

   Apply the formula once to help evaluate

$$\int_0^{\pi/2} \sin^n(x)\,dx.$$

3. Recall that the Fibonacci sequence is defined *recursively* by $F_n = F_{n-1} + F_{n-2}$. Consider a sequence $\{I_n\}$, where

$$I_n = \int_0^{\pi/2} \sin^n(x)\,dx.$$

   Develop a recursive definition for I_n.

4. You are looking for patterns in this problem. Leave your answers as expressions. (Don't do the arithmetic.)

   Given that $I_n = \dfrac{n-1}{n} I_{n-2}$ express:

   (a) I_2 in terms of I_0 (b) I_4 in terms of I_0 (c) I_6 in terms of I_0
   (d) I_{12} in terms of I_0 (e) I_{2n} in terms of I_0

   (f) I_3 in terms of I_1 (g) I_5 in terms of I_1 (h) I_7 in terms of I_1
   (i) I_{13} in terms of I_1 (j) I_{2n+1} in terms of I_1

5. Find exact values for I_0 and I_1.

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