

Average Value

1. With your group, write a definition of *average value*. Keep your definition handy when you answer the next 5 questions.
2. What is the *average value* of the numbers in this list:

1, 2, 2, 3

3. How might you compute the *average value* of the temperature at the Lowell flag pole over a 24 hour period?

4.

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1, \\ 2 & \text{if } 1 < x \leq 3, \\ 3 & \text{if } 3 < x \leq 4. \end{cases}$$

Draw a graph of f for $x \in [0, 4]$. What is the *average value* of $f(x)$ for $x \in [0, 4]$.

5. Draw a graph of $\sin(x)$ for $x \in [0, \pi/2]$. What is the *average value* of $\sin(x)$ for $x \in [0, \pi/2]$?
6. With your group, revise your definition of *average value* to take into account your responses to questions 2-5. Think about both algebraic and geometric ways to view *average value*.

Euler's Method I

7. Consider the DE $y' = xy$ with initial value $y(0) = 1$.
For each use of Euler's method, be sure to set up a table with column headings:

step	x	y	y'	Δy
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The first data row of your table, *step 0*, contains the initial x and y values as well as computed values for y' and Δx .

- (a) Use Euler's method with four steps of size 0.25 to estimate $y(1)$

Two Homework Questions – TurnIn Tuesday March 22

8. **Euler's Method II** Consider the DE $y' = xy$ with initial value $y(0) = 1$.

For each use of Euler's method, be sure to set up a table with column headings:

step	x	y	y'	Δy
------	-----	-----	------	------------

The first data row of your table, *step 0*, contains the initial x and y values as well as computed values for y' and Δx .

- Use Euler's method with one step of size 1 to estimate $y(1)$.
- Use Euler's method with two steps of size 0.5 to estimate $y(1)$
- Use Euler's method with four steps of size 0.25 to estimate $y(1)$
- Use Euler's method with eight steps of size 0.125 to estimate $y(1)$
- It turns out that you can actually solve this IVP (Initial Value Problem). Solve for $y(x)$. (Be sure to check your answer.)
- Compare the exact value of $y(1)$ with the estimates your group computed in parts (a)-(d). Make a table with column headings

Steps	estimated $y(1)$	error
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where the *error* is the difference between the *estimated* $y(1)$ and the *exact* $y(1)$ to three decimal places.

- Explain how the results of part (f) suggest that the error made by Euler's method is inversely proportional to n , where n is the number of steps.

9. Growing at Half Capacity

The *logistic growth model* is a model of population P versus time t given by the following differential equation: $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$. In this exercise, we are concerned with the moment when the population P is equal to half the carrying capacity ($P = M/2$).

- Show that if the initial population is P_0 , then $P = M/2$ when the time t is equal to

$$T = \frac{1}{k} \ln \frac{M - P_0}{P_0}$$

- What does $\frac{d^2P}{dt^2}$ represent in physical terms?
- Show that the population is growing fastest at time T .

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Two Homework Questions – TurnIn Tuesday March 22

8. **Euler's Method II** Consider the DE $y' = xy$ with initial value $y(0) = 1$.

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