1. Consider the DE $y' = xy$ with initial value $y(0) = 1$.
   For each use of Euler's method, be sure to set up a table with column headings:

step	x	y	y'

   (a) Use Euler's method with one step of size 1 to estimate $y(1)$.

   (b) Use Euler's method with two steps of size 0.5 to estimate $y(1)$

   (c) Use Euler's method with four steps of size 0.25 to estimate $y(1)$

   (d) Use Euler's method with eight steps of size 0.125 to estimate $y(1)$

   (e) It turns out that you can actually solve this IVP (Intial Value Problem). Solve for $y(x)$. (Be sure to check your answer.)

   (f) Compare the exact value of $y(1)$ with the estimates your group computed in parts (a)-(d). Make a table with column headings

Steps	estimated $y(1)$	error

   where the *error* is the difference between the *estimated* $y(1)$ and the *exact* $y(1)$ to three decimal places.

   (g) Explain how the results of part (f) suggest that the error made by Euler's method is inversely proportional to n, where n is the number of steps.

2. **Growing at Half Capacity**

   The *logistic growth model* is a model of population P versus time t given by the following differential equation: $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$. In this exercise, we are concerned with the moment when the population P is equal to half the carrying capacity ($P = M/2$).

   (a) Show that if the initial population is P_0, then $P = M/2$ when the time t is equal to

   $$T = \frac{1}{k} \ln \frac{M - P_0}{P_0}$$

   (b) What does $\frac{d^2P}{dt^2}$ represent in physical terms?

   (c) Show that the population is growing fastest at time T.

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