1. Find the Taylor series for $f(x) = \ln(x)$ when $a = 1$.

   (a) Demonstrate how to do this by *brute force*, taking the derivatives of $\ln(x)$ at $x = 1$.

   (b) Demonstrate how to do this starting with the relationship of $f(x) = \dfrac{1}{1-x}$ and a simple converging geometric series.

2. If $f(x)$ is an odd function then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

   (a) Draw a graph and explain why.

   (b) Use the algebraic definition of an odd function to explain why.
   Game Plan: Consider $G(x) = \displaystyle\int_0^x f(t)\,dt$.

      i. What do you need to show about $G(x)$?

      ii. Set up (but don't evaluate) a u−substitution where $u(t) = -t$.

      iii. Put the parts (i) and (ii) together.

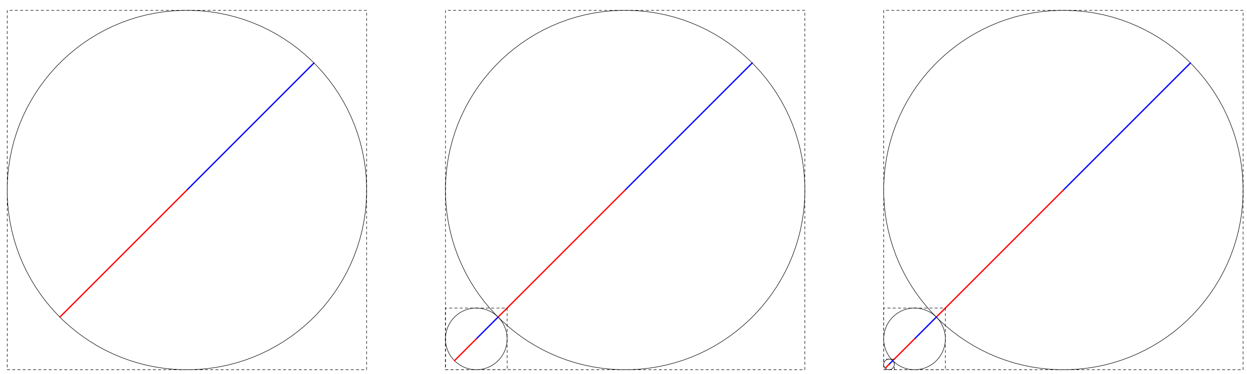
3. Assume the power series

$$p(x) = x - 4x^3 + 16x^5 - 64x^7 + 256x^9 + \ldots$$

   is a Maclaurin series for some function $f(x)$. Write an equation (a simple expression) for $f(x)$.

4. Reconsider the *Snuggly Circle Problem* using this idea:

   A third circle (C_3) can snuggly fit in the space between C_2 and the lower left hand corner of the square. Continue fitting in a sequence of circles, C_k. Use this sequence to determine the radius of C_2. [Hint: consider the sequence of diameters.]



5. Suppose we have a population of bacteria in a petri dish. We want to model the number of bacteria as a function of time, $y(t)$. Assume at time 0, y_0 bacteria are in the dish. Also assume the dish can only hold M bacteria.

   (a) Explain why the differential equation, $\dfrac{dy}{dt} = ky\left(1 - \dfrac{y}{M}\right)$, makes sense for this situation.

   (b) Find an equation for $y(t)$ in terms of k, M, y_0, and t. Be sure your work is easy to follow.

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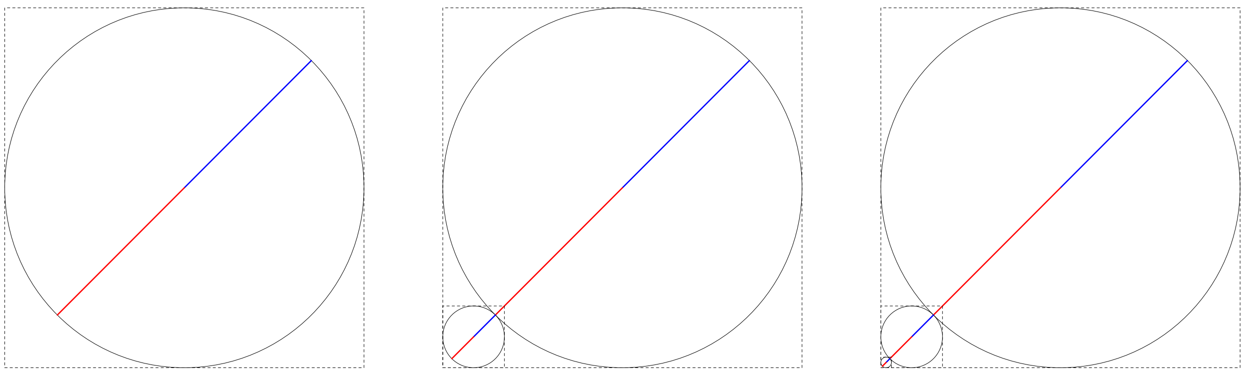

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