

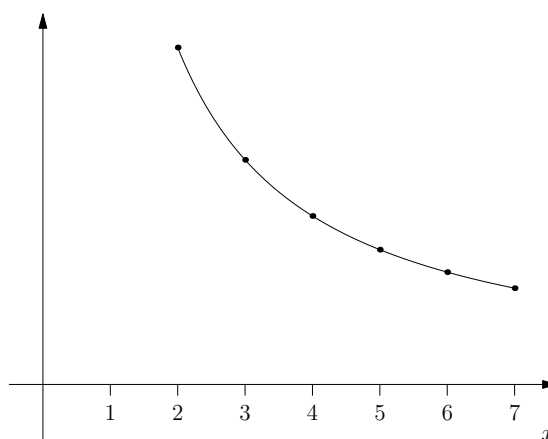
1. **DoNow:** Concentric Circles Problem

Given a pair of concentric circles, a chord of length $2a$ in the bigger circle is tangent to the smaller circle. Do you have enough information to find the area of the region between the two circles? If so find the area, if not what other information do you need?

2. Use a graph paper white board to show:

$$\int_2^6 \frac{10}{x} dx > \sum_{k=3}^6 \frac{10}{k} > \int_3^7 \frac{10}{x} dx$$

Use colors and write an explanation.



3. A Fibonacci sequence is defined recursively as $f_1 = 1, f_2 = 1, f_n = f_{n-1} + f_{n-2}$.

(a) Write out the first 7 terms of the sequence.

(b) Now consider the sequence $\{r_k\}$ where $r_k = \frac{f_{k+1}}{f_k}$. Write out the first 7 terms of this sequence.

(c) Should r_n converge as $n \rightarrow \infty$? If so, find the limit. If not, explain how you know.

4. If possible, we want to find the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$

- (a) Consider the recursive sequence $a_0 = 0$, $a_{n+1} = \sqrt{1 + a_n}$. Compute the next five terms a_1 , a_2 , a_3 , a_4 , and a_5 . (You can use a calculator.)
- (b) Were any of the values of a_k in the part (a) greater than 2?
- (c) Explain how you can tell that $a_n < 2$ for all n .
- (d) How do you know that $a_{n+1} > a_n$?
- (e) Since $\{a_n\}$ is increasing and bounded above by 2, the Monotone Sequence Theorem says that $\{a_n\}$ converges. If $\lim_{n \rightarrow \infty} a_n = a$, show that $a = \sqrt{1 + a}$.

(f) What is the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$?

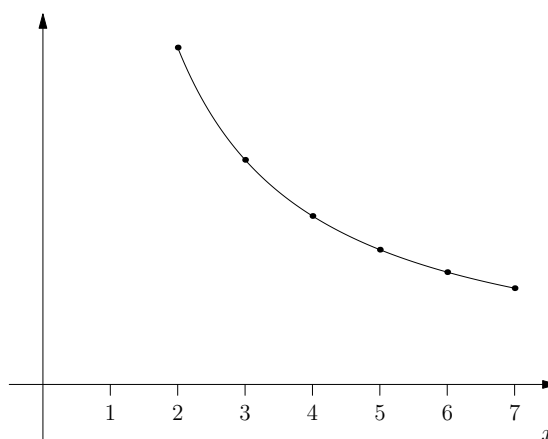
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