

1. **DoNow:** The Random Decimal

- (a) Have each person in your group think of a random integer from 0 through 9. Let  $a_1$  be  $0.wxyz$  where  $w, x, y$  and  $z$  are your numbers. For example, if you came up with 2, 4, 1, and 8, then you would write  $a_1 = 0.2418$ .
- (b) Have each person in your group think of a new integer, and add those integers to the end of  $a_1$  to form  $a_2$ . For example, if you already had  $a_1 = 0.2418$ , you might come up with  $a_2 = 0.24185299$ . Continue the process to form  $a_3, a_4$  and  $a_5$ .
- (c) If you continued this process infinitely many times, you would have an infinite sequence  $\{a_n\}$ . Does this sequence converge, diverge, or is it impossible to tell? Why?

2. Recall we decided that the differential equation  $\frac{dy}{dt} = -k(y - 20)$  could be used to model the temperature of a cup of hot coffee as it cools where room temperature is 20 degrees. Solve this DE when  $y(0) = 100$  degrees. Be sure to check your answer.

3. A Fibonacci sequence is defined recursively as  $f_1 = 1, f_2 = 1, f_n = f_{n-1} + f_{n-2}$ .

- (a) Write out the first 7 terms of the sequence.
- (b) Now consider the sequence  $\{r_k\}$  where  $r_k = \frac{f_{k+1}}{f_k}$ . Write out the first 7 terms of this sequence.
- (c) Should  $r_n$  converge as  $n \rightarrow \infty$ ? If so, find the limit. If not, explain how you know.

4. If possible, we want to find the value of  $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$

- (a) Consider the recursive sequence  $a_0 = 0, a_{n+1} = \sqrt{1 + a_n}$ . Compute the next five terms  $a_1, a_2, a_3, a_4,$  and  $a_5$ . (You can use a calculator.)
- (b) Were any of the values of  $a_k$  in the part (a) greater than 2?
- (c) Explain how you can tell that  $a_n < 2$  for all  $n$ .
- (d) How do you know that  $a_{n+1} > a_n$ ?
- (e) Since  $\{a_n\}$  is increasing and bounded above by 2, the Monotone Sequence Theorem says that  $\{a_n\}$  converges. If  $\lim_{n \rightarrow \infty} a_n = a$ , show that  $a = \sqrt{1 + a}$ .

(f) What is the value of  $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$ ?

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