1. Evaluate $\int \sin^2(\theta)\, d\theta$. Demonstrate two different approaches

2. Evaluate $\int_0^1 \sqrt{1-x^2}\, dx$.

3. Now, our goal is to get a general antiderivative for $\int \sqrt{1-x^2}\, dx$.

   (a) Look at the triangle in the graph at the right. Write an equation to express x in terms of θ. Explain.

   $f(x) = \sqrt{1-x^2}$

   $(x, f(x))$

   θ

   x

   (b) Consider what happens when you do a θ-substitution by letting $x = \cos\theta$. Push through (e.g. $dx = \ldots$) and get an expression for I in terms of x.

   (c) Check your answer.
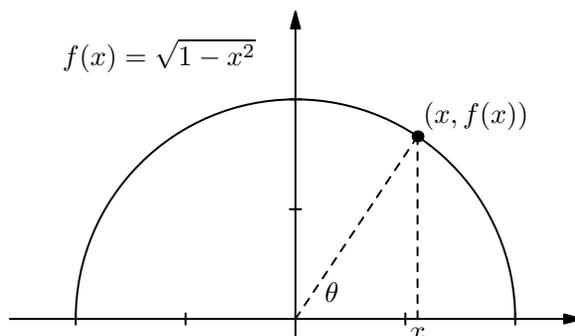
Followup Questions

4. *Napkin Ring Problem – Revisited:* You discovered that the volume of a napkin ring is independent of radius of the original sphere. The volume is simply a function of h, the height of the napkin ring.

   h
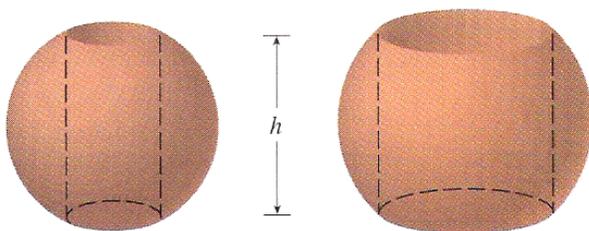
   You explain this conclusion to our inquisitive eighth grader without telling her the formula for the napkin ring volume. She replies "that's cool! Let me see if I can use that fact to come up with a formula for $V(h)$, the volume of a napkin ring."

   After giving the problem some thought, she comes back to tell you an equation for $V(h)$ and explains how she solved the problem without using calculus.

   Demonstrate how she might have done this.

5. Consider the graph of $f(x) = \sqrt{1 - x^2}$. Confirm your understanding of using integrals to compute arc length (yes, *Particle, the Dog*) by finding the length of the graph of $f(x)$ for $x \in [0, 1]$. How can you check your answer?

6. Use θ-substitution on $\int \sqrt{a^2 - x^2}\, dx$.

   (a) Get an antiderivative. Be sure to check your result.

   (b) Use your result to evaluate $\int_1^2 \sqrt{a^2 - x^2}\, dx$.

7. *More on the Napkin Ring*

   Consider slicing the napkin ring with height, h, and sphere radius, R, into washers. Setup a volume element, dV. Can you show that R does not matter by just considering the equation for dV without evaluating the integral? Explain.

Homework Due: Tuesday 1/17/17

Turn In: Two *Followup Question* write-ups, each should be on its own page. Each should fit on a single side.

   (a) one from problem 4 or 7 (*Napkin Ring problems*)

   (b) one from problem 5 or 6 (arc length or θ-substitution)