**Homework Problems**

Assume that all functions in these questions are differentiable. The overall goal of these problems is to use the difference quotient to develop three fundamental rules and one key short cut to establish an easy procedure for evaluating the derivatives of polynomials. Use stratch paper (or another copy of this worksheet) for your first drafts.

1. If $f(x) = x$, then evaluate $f'(x)$. Explain

2. Recall the limit rules (e.g. *the limit of a product is the product of the limits*). In English, write each proposed derivative rule, and use a difference quotient to show:

   (a) $(k \cdot f(x))' = k \cdot f'(x)$ (where $k$ is a constant)

   (b) $(f(x) + g(x))' = f'(x) + g'(x)$

3. If $f(x)$ is differentiable and $g(x) = x \cdot f(x)$, use the definition of a derivative to show $g'(x) = x \cdot f'(x) + f(x)$

4. Using *only* the results of the previous questions, evaluate the derivatives of each of these functions. This means you don't use a difference quotient (and certainly don't use rules you may have learned outside of this class). Be sure to record your steps and briefly explain your method.

   (i) $f(x) = x^2$ (ii) $g(x) = x^3$ (iii) $h(x) = x^4$

(continued on other side)

4. (Continued) Using *only* the results of the previous questions, evaluate the derivatives of each of these functions. This means you don't use a difference quotient (and certainly don't use rules you may have learned outside of this class). Be sure to record your steps and a brief explain your method.

   (iv) $j(x) = x^5$ (v) $k(x) = x^7$ (vi) $l(x) = x^{10}$

5. If you did question 4 correctly, you can observe the pattern that when $f(x) = x^k$, $f'(x) = k \cdot x^{k-1}$. At a minimum, this works for all positive integers less than or equal to 10. The pattern could continue well past 10. Let's assume that the pattern holds for all positive integers up to (at least) $n$.
   Given that $(x^n)' = n \cdot x^{n-1}$, show $(x^{n+1})' = (n+1) \cdot x^n$.

6. Assume $f(x) = 2x^{400} - 3x^{150} + 9$. Find $f'(x)$. Explain your method.

7. Rewrite $x^4 + ax^3 + a^2x^2 + a^3x + a^4$ using sigma notation.