Geometry of Complex Multiplication

- Multiplication by a real number:

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- Dilation (stretching)

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- Dilation (stretching)
- Multiplication by i as a 90° rotation

Taylor Expansion of e^x

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Taylor Expansion of e^x

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...$$

$$= 1 + 1(x) + x\left(\frac{x}{2}\right) + \frac{x^2}{2}\left(\frac{x}{3}\right) + \frac{x^3}{3!}\left(\frac{x}{4}\right) + ...$$

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$$e^x = t_0 + t_1 + t_2 + ...$$

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Explicitly: $t_n = \frac{x^n}{n!}$

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Explicitly: $t_n = \frac{x^n}{n!}$

Recursively: $t_0 = 1$

Taylor Expansion of e^x

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Explicitly: $t_n = \frac{x^n}{n!}$

Recursively: $t_0 = 1;$

$$t_n = \frac{x^n}{n!}$$

Taylor Expansion of e^x

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$$e^x = t_0 + t_1 + t_2 + ...$$

Explicitly: $t_n = \dfrac{x^n}{n!}$

Recursively: $t_0 = 1;$

$$t_n = \frac{x^n}{n!} = \left(\frac{x^{n-1}}{(n-1)!}\right) \cdot \frac{x}{n}$$

Taylor Expansion of e^x

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...$$

$$= 1 + 1(x) + x\left(\frac{x}{2}\right) + \frac{x^2}{2}\left(\frac{x}{3}\right) + \frac{x^3}{3!}\left(\frac{x}{4}\right) + ...$$

$$e^x = t_0 + t_1 + t_2 + ...$$

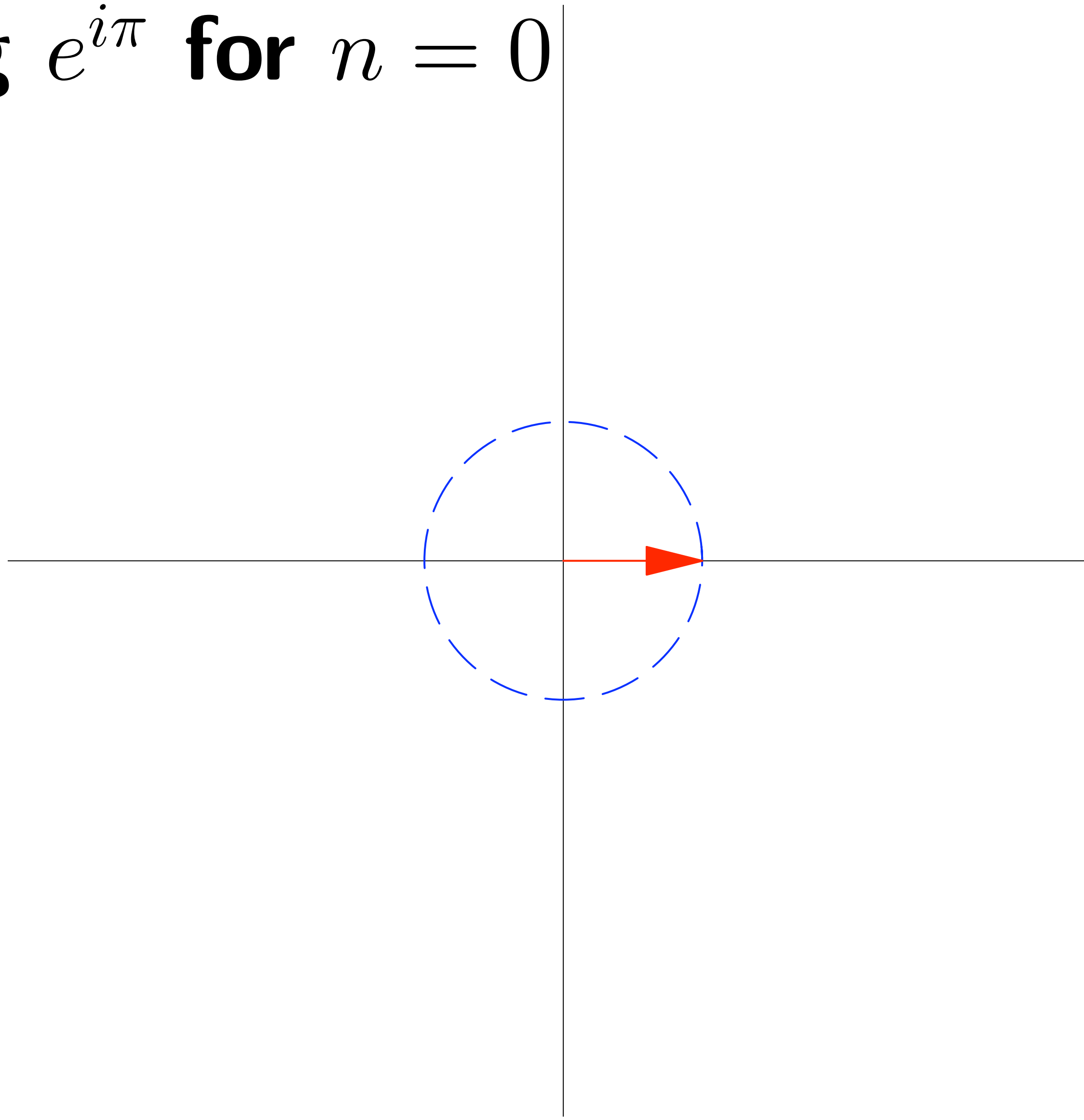
Explicitly: $t_n = \dfrac{x^n}{n!}$

Recursively: $t_0 = 1;$

$$t_n = \frac{x^n}{n!} = \left(\frac{x^{n-1}}{(n-1)!}\right) \cdot \frac{x}{n} = t_{n-1} \cdot \frac{x}{n}$$

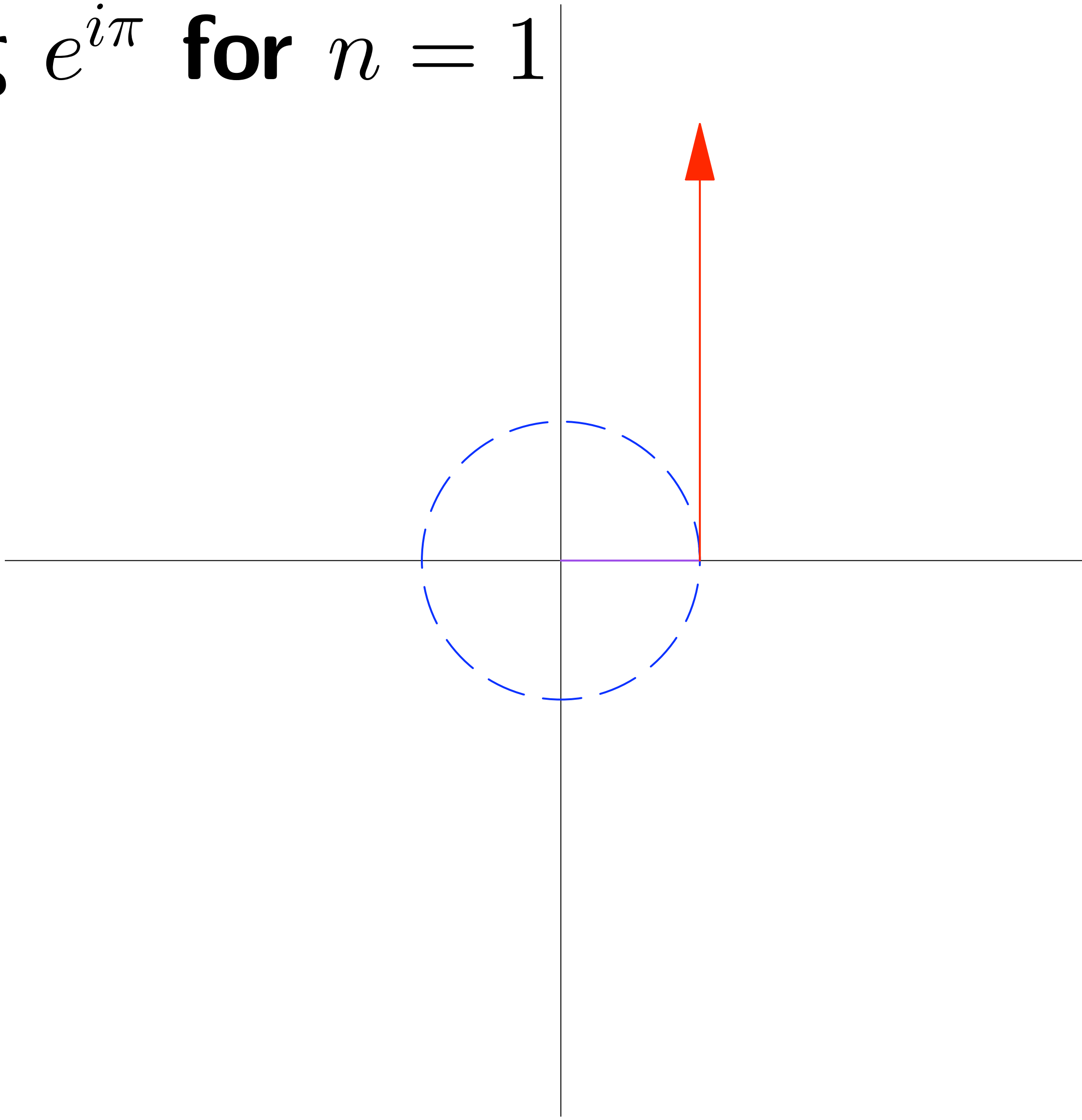
Expanding $e^{i\pi}$ for $n = 0$

1



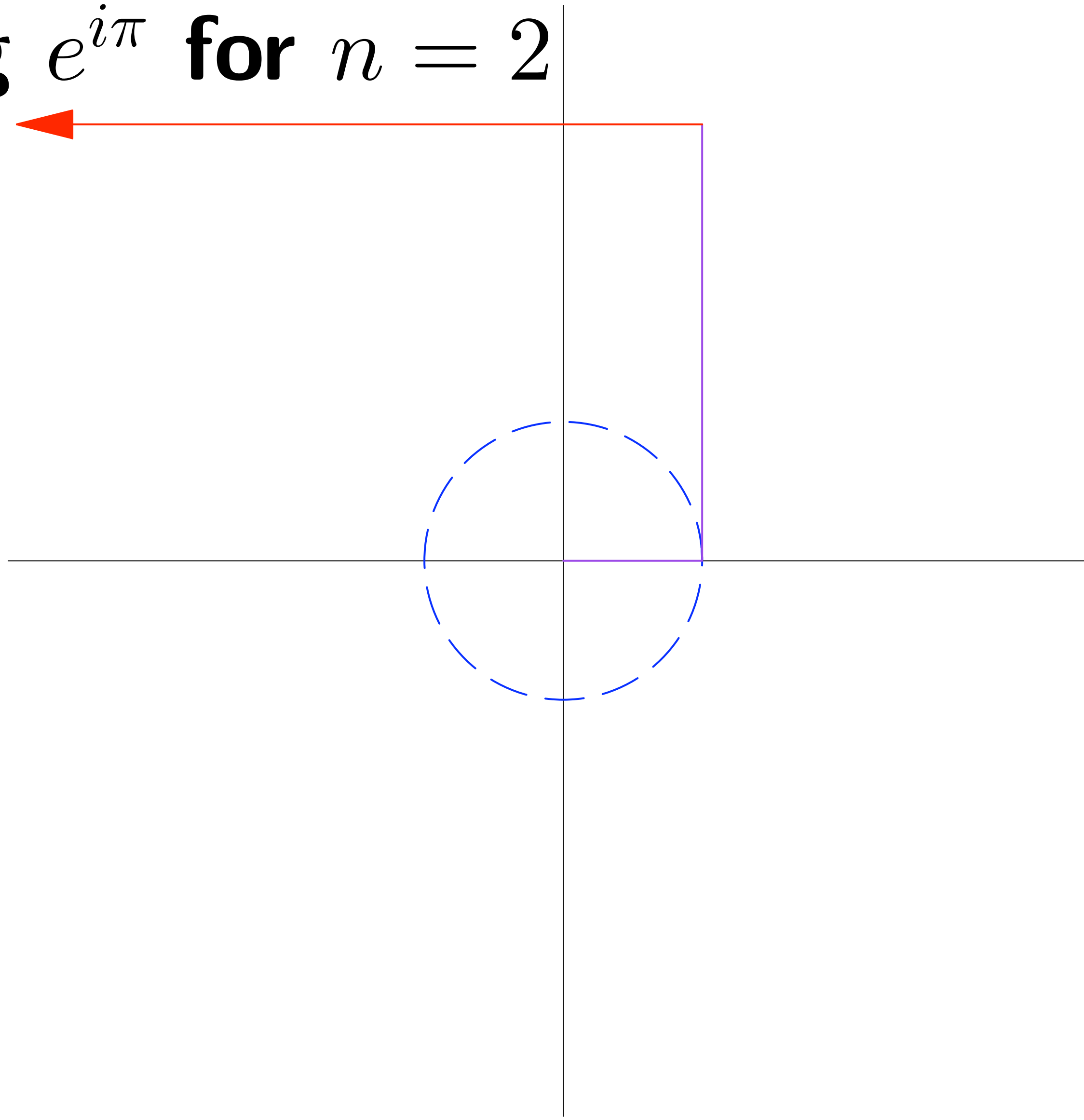
Expanding $e^{i\pi}$ for $n = 1$

$1 + i\pi$



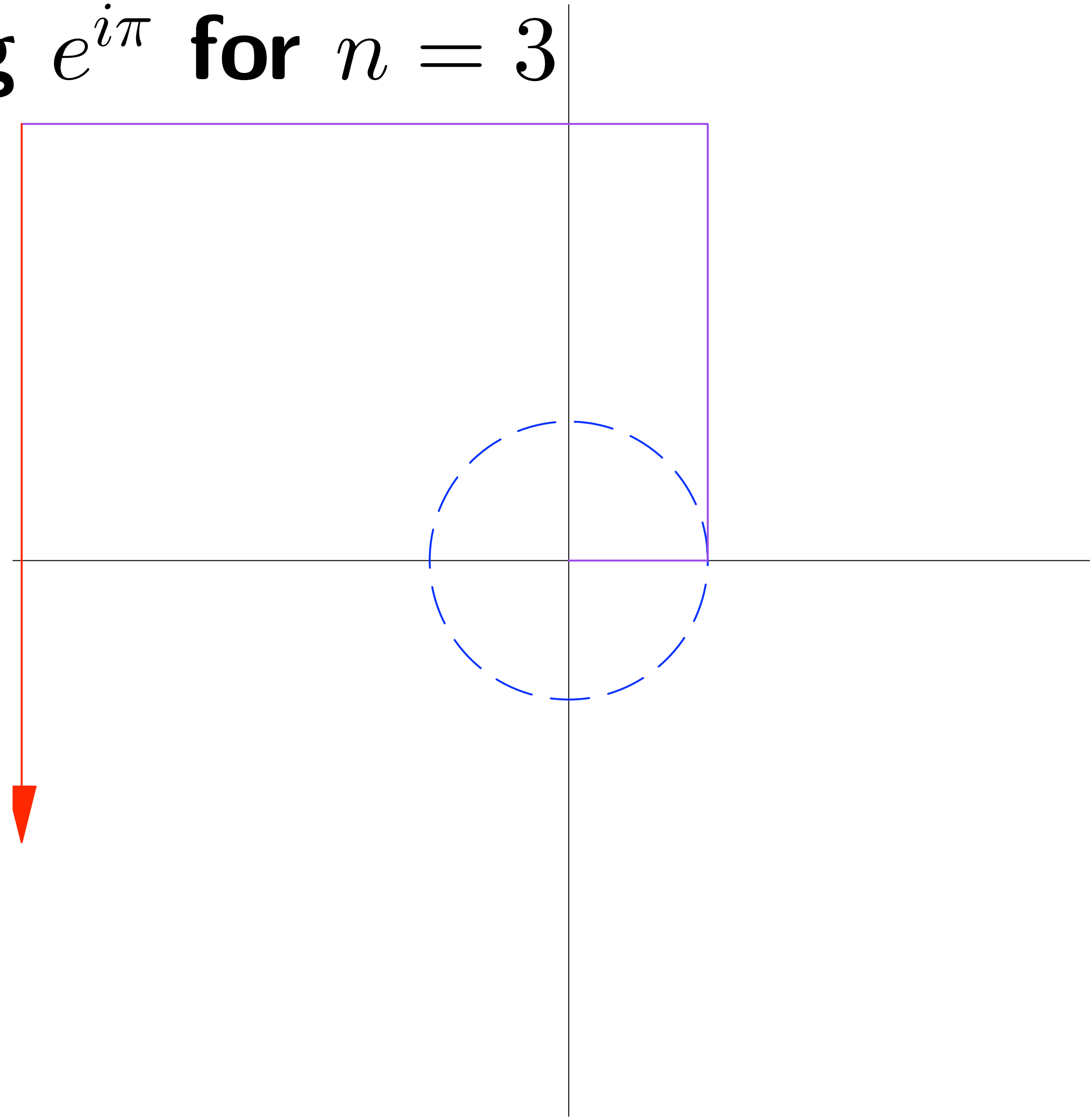
Expanding $e^{i\pi}$ for $n = 2$

$$1 + i\pi + i\frac{\pi}{2}i\pi$$



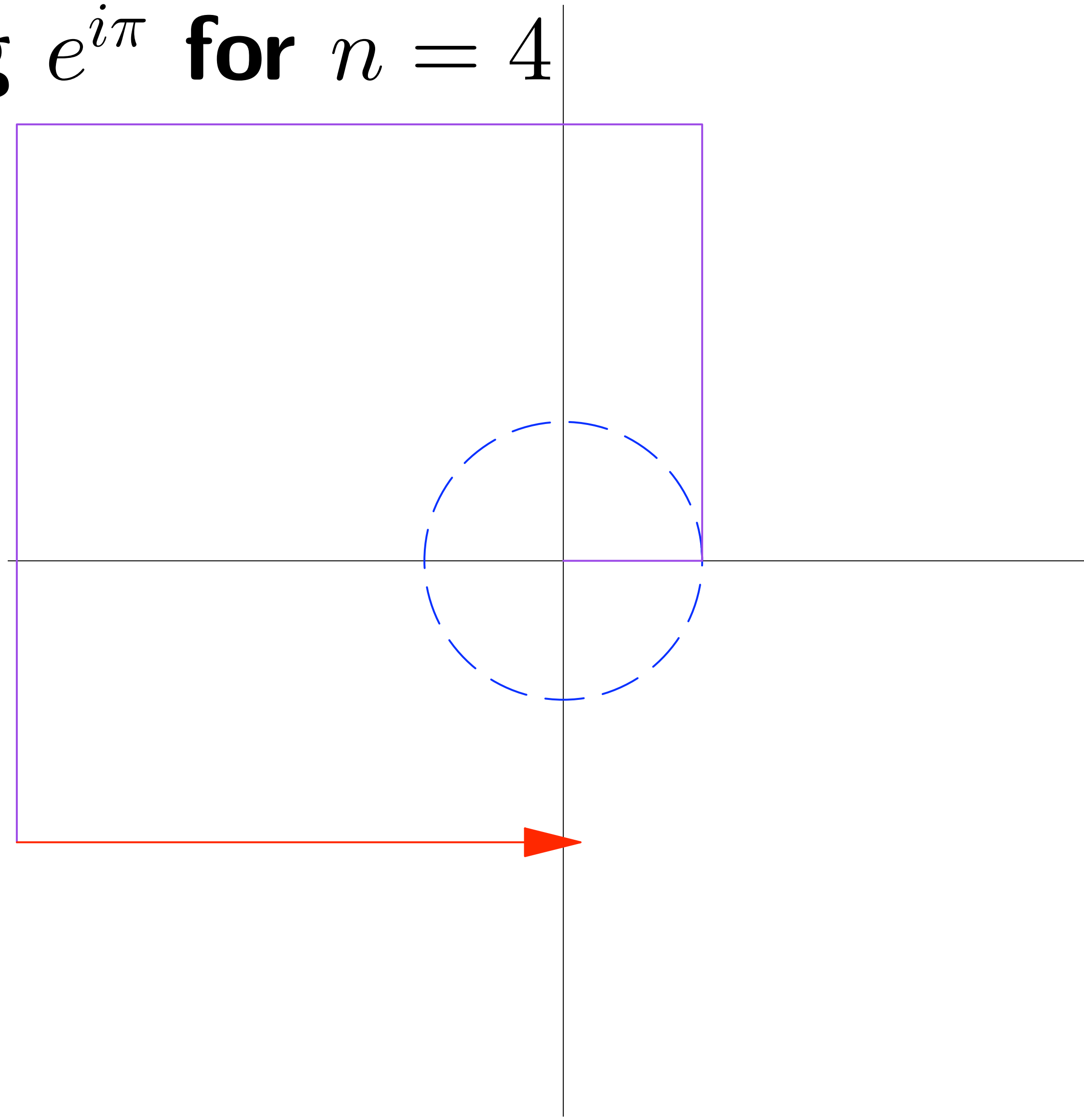
Expanding $e^{i\pi}$ for $n = 3$

$$\sum_{k=0}^{2} \frac{(i\pi)^k}{k!} + i\frac{\pi}{3}\frac{(i\pi)^2}{2!}$$



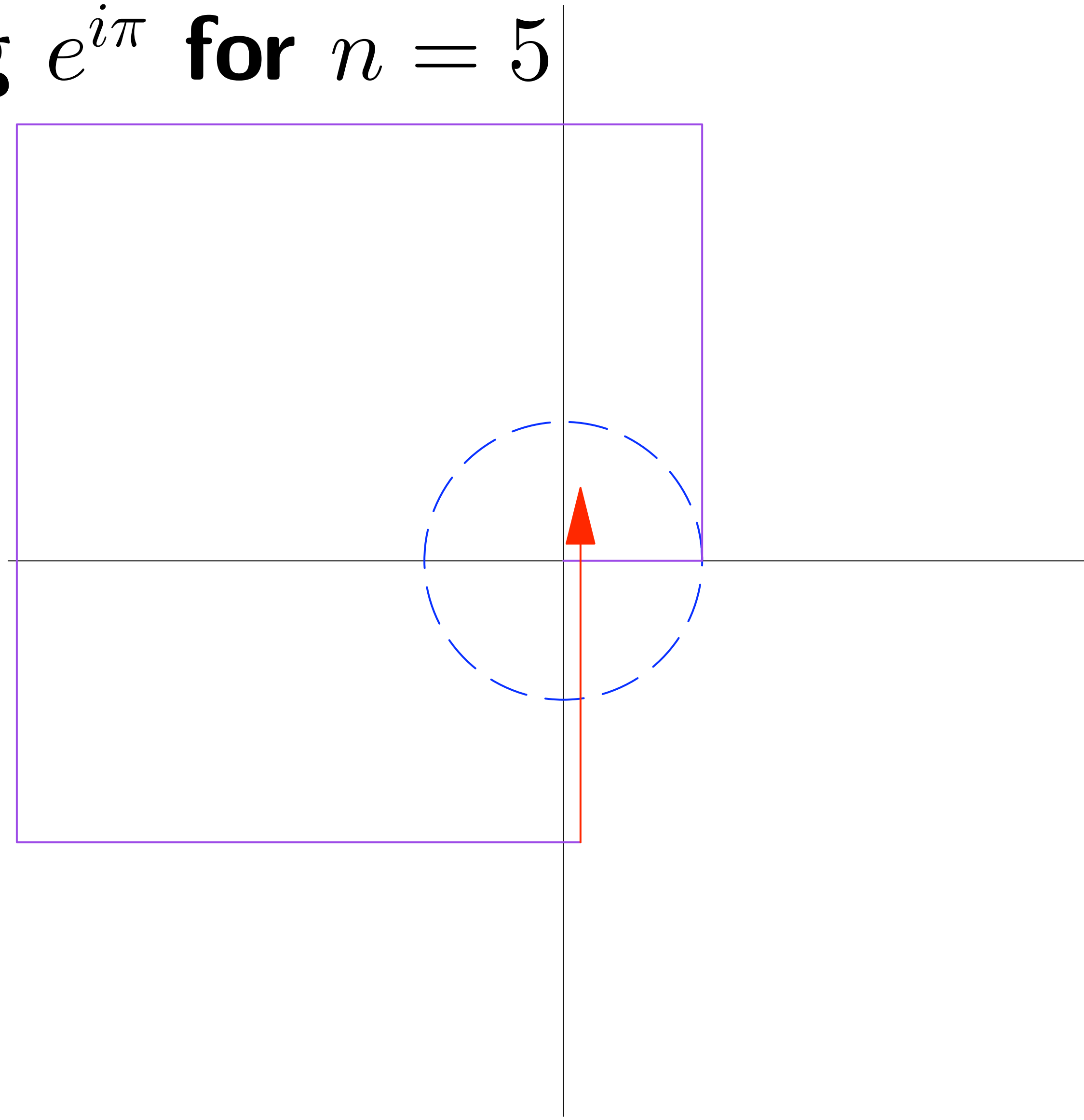
Expanding $e^{i\pi}$ for $n = 4$

$$\sum_{k=0}^{3} \frac{(i\pi)^k}{k!} + i\frac{\pi}{4}\frac{(i\pi)^3}{3!}$$



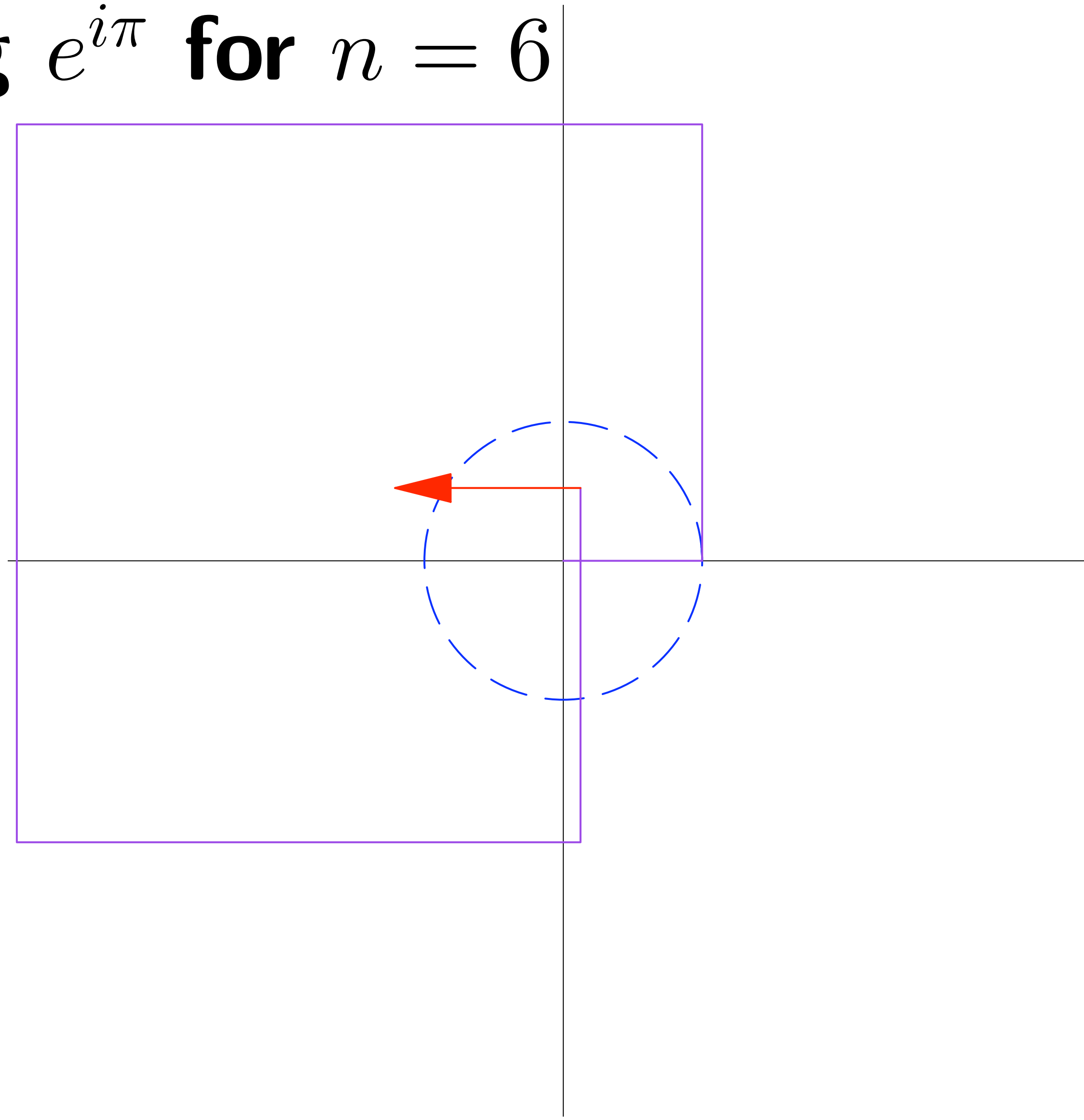
Expanding $e^{i\pi}$ for $n = 5$

$$\sum_{k=0}^{4} \frac{(i\pi)^k}{k!} + i\frac{\pi}{5}\frac{(i\pi)^4}{4!}$$



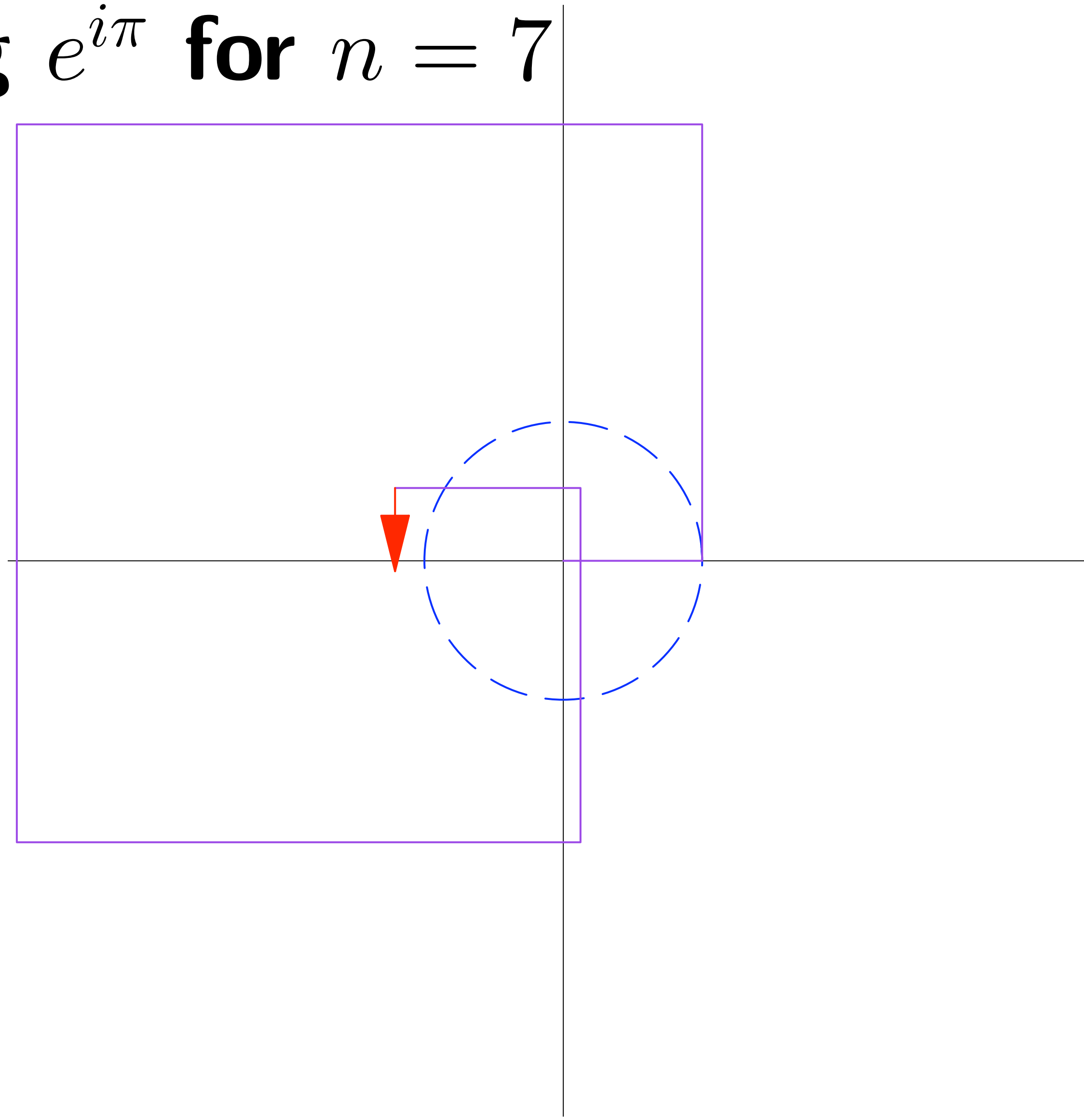
Expanding $e^{i\pi}$ for $n = 6$

$$\sum_{k=0}^{5} \frac{(i\pi)^k}{k!} + i\frac{\pi}{6}\frac{(i\pi)^5}{5!}$$



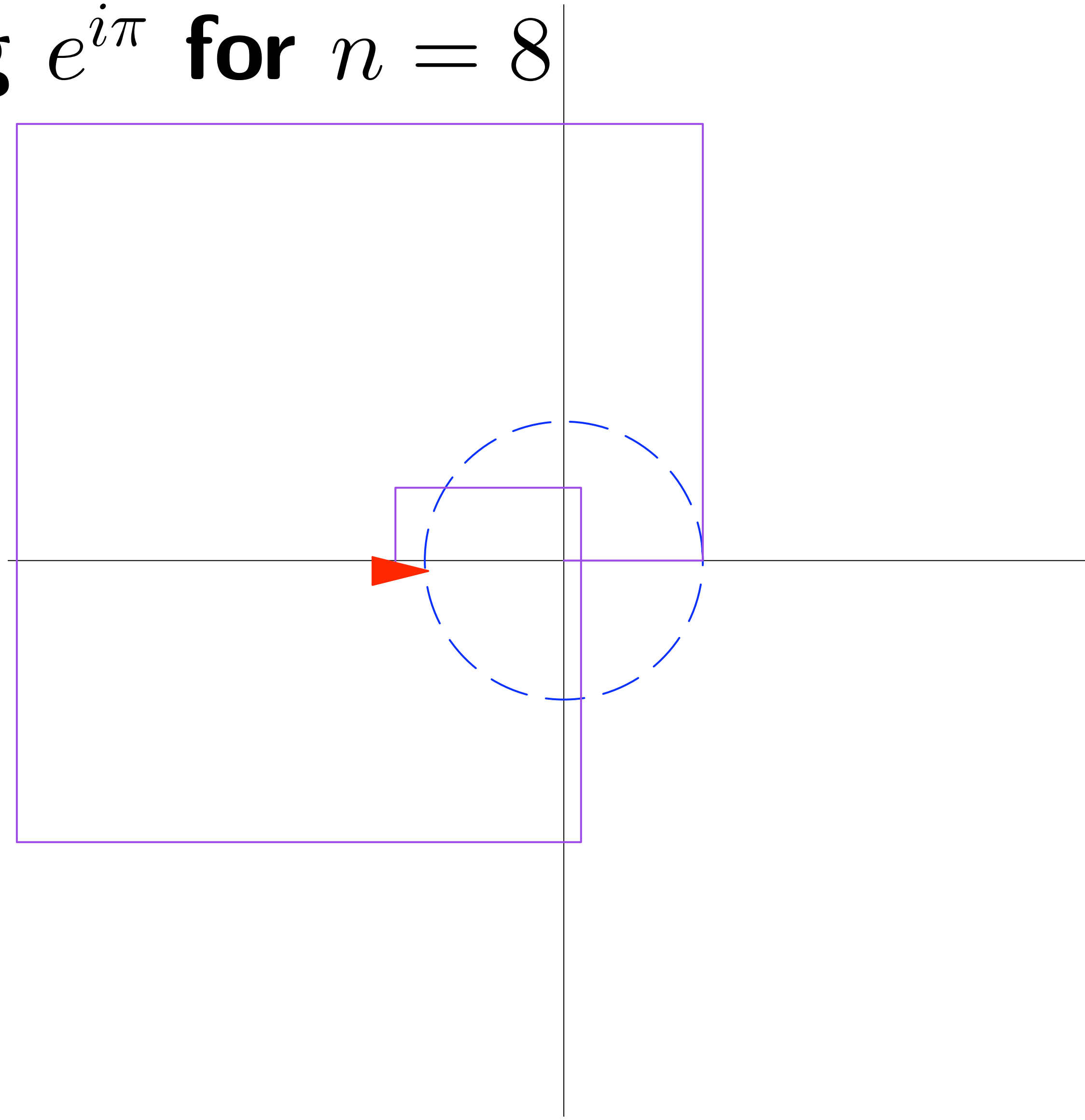
Expanding $e^{i\pi}$ for $n = 7$

$$\sum_{k=0}^{6} \frac{(i\pi)^k}{k!} + i\frac{\pi}{7}\frac{(i\pi)^6}{6!}$$



Expanding $e^{i\pi}$ for $n = 8$

$$\sum_{k=0}^{7} \frac{(i\pi)^k}{k!} + i\frac{\pi}{8}\frac{(i\pi)^7}{7!}$$



Expanding $e^{i\pi}$ for $n = 9$

$$\sum_{k=0}^{8} \frac{(i\pi)^k}{k!} + i\frac{\pi}{9}\frac{(i\pi)^8}{8!}$$