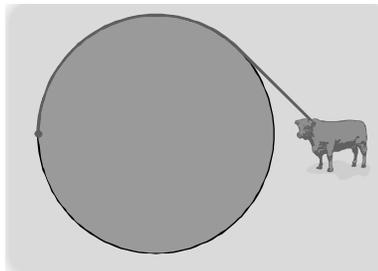


Clara Cowculus

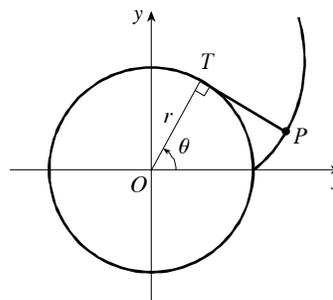
Clara the Calculus Cow has been tied to a silo with radius r by a rope just long enough to reach a point diametrically opposite to the point where she is tied, as depicted in the diagram.



If she goes to the left side of the silo, she can stand far away from the silo, while at the right side, she can only graze right next to the silo. We wish to compute the total area of the region upon which she can graze.

- How far from the silo can sweet Clara stand when she is to its left?
- Draw a picture of the shape of the region that she can reach.
- Now assume that a very long rope is wound around the circular silo, and then unwound while being held taut.

The curve traced by the end of the rope is called the involute of the circle. If the silo has radius r and center O , as shown in the figure below, and if the parameter θ is chosen as in the figure, find the parametric equations of the involute.



- Use Desmos to draw the region sketched in part b.
- What is the area of the grazing region available to Clara the Calculus Cow? The involute is a parametric curve. Begin by developing a typical rectangular area element.