Using FTC-2

Now we know that to solve certain kinds of problems, those that lead to a sum of a certain form, we "merely" find an antiderivative and substitute two values and subtract. Unfortunately, finding antiderivatives can be quite difficult. While there are a small number of rules that allow us to compute the derivative of any common function, there are no such rules for antiderivatives. There are some techniques that frequently prove useful, but we will never be able to reduce the problem to a completely mechanical process.

Because of the close relationship between an integral and an antiderivative, the integral sign is also used to mean "antiderivative". You can tell which is intended by whether the limits of integration are included:

$$\int_1^2 x^2\,dx$$

is an ordinary integral, also called a **definite integral**, because it has a definite value, namely

$$\int_1^2 x^2\,dx = \frac{2^3}{3} - \frac{1^3}{3} = \frac{7}{3}.$$

We use

$$\int x^2\,dx$$

to denote the antiderivative of x^2, also called an **indefinite integral**. So this is evaluated as

$$\int x^2\,dx = \frac{x^3}{3} + C.$$

It is customary to include the constant C to indicate that there are really an infinite number of antiderivatives. We do not need this C to compute definite integrals, but in other circumstances we will need to remember that the C is there, so it is best to get into the habit of writing the C. When we compute a definite integral, we first find an antiderivative and then substitute. It is convenient to first display the antiderivative and then do the substitution; we need a notation indicating that the substitution is yet to be done. A typical solution would look like this:

$$\int_1^2 x^2\,dx = \left.\frac{x^3}{3}\right|_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{7}{3}.$$

The vertical line with subscript and superscript is used to indicate the operation "substitute and subtract" that is needed to finish the evaluation.

FTC-2

THEOREM 7.2.1 Fundamental Theorem of Calculus Suppose that $f(x)$ is continuous on the interval $[a, b]$. If $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Exercises 7.2.

Find the antiderivatives of the functions:

1. $8\sqrt{x}$ ⇒

2. $3t^2 + 1$ ⇒

3. $4/\sqrt{x}$ ⇒

4. $2/z^2$ ⇒

5. $7s^{-1}$ ⇒

6. $(5x+1)^2$ ⇒

7. $(x-6)^2$ ⇒

8. $x^{3/2}$ ⇒

9. $\dfrac{2}{x\sqrt{x}}$ ⇒

10. $|2t-4|$ ⇒

Compute the values of the integrals:

11. $\displaystyle\int_1^4 t^2 + 3t\,dt$ ⇒

12. $\displaystyle\int_0^\pi \sin t\,dt$ ⇒

13. $\displaystyle\int_1^{10} \frac{1}{x}\,dx$ ⇒

14. $\displaystyle\int_0^5 e^x\,dx$ ⇒

15. $\displaystyle\int_0^3 x^3\,dx$ ⇒

16. $\displaystyle\int_1^2 x^5\,dx$ ⇒

17. Find the derivative of $G(x) = \displaystyle\int_1^x t^2 - 3t\,dt$ ⇒

18. Find the derivative of $G(x) = \displaystyle\int_1^{x^2} t^2 - 3t\,dt$ ⇒

19. Find the derivative of $G(x) = \displaystyle\int_1^x e^{t^2}\,dt$ ⇒

20. Find the derivative of $G(x) = \displaystyle\int_1^{x^2} e^{t^2}\,dt$ ⇒

21. Find the derivative of $G(x) = \displaystyle\int_1^x \tan(t^2)\,dt$ ⇒

22. Find the derivative of $G(x) = \displaystyle\int_1^{x^2} \tan(t^2)\,dt$ ⇒