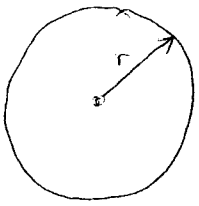
Example 1:

The radius of a circular puddle is 3 meter and it is increasing at the rate of 1 cm/min. How fast is the puddle's area increasing?

GENERAL SITUATION

1) 

2) $A = \pi r^2$

3) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

SPECIFIC SITUATION

4) $r = 300\text{ cm}$, $\frac{dr}{dt} = 1\ \frac{\text{cm}}{\text{min}}$

$\frac{dA}{dt} = ?$

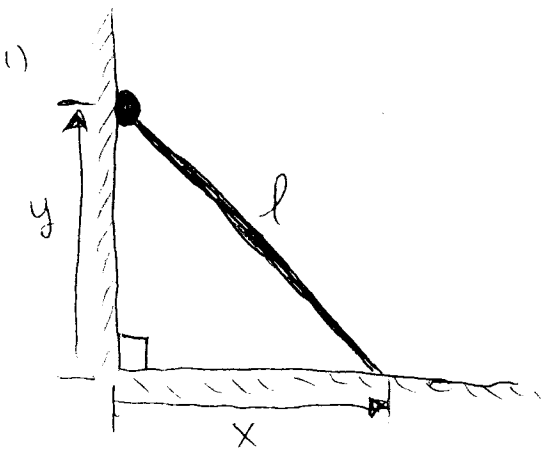
5) $\frac{dA}{dt} = 2\pi(300\text{ cm})\left(1\ \frac{\text{cm}}{\text{min}}\right)$

$\Rightarrow \boxed{\frac{dA}{dt} = 600\pi\ \frac{\text{cm}^2}{\text{min}}}$

Example 2:

A bag is tied to the top of a 5 meter long ladder which is resting against a vertical wall. Suppose the ladder begins sliding down the wall in such a way that the foot of the ladder is moving away from the wall. How fast is the bag descending at the instant the foot of the ladder is 4 meters from the wall and the foot of the ladder is moving away from the wall at the rate of 2 meters/sec?

GENERAL SITUATION

1) 

2) $x^2 + y^2 = \ell^2$

3) $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

SPECIFIC SITUATION

4) $x = 4\text{m} \Rightarrow (4)^2 + y^2 = 5^2$

$\Rightarrow y = 3\text{m}$

$\frac{dx}{dt} = 2\ \frac{\text{m}}{\text{sec}}$ $\qquad \frac{dy}{dt} = ?$

5) $\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

$\Rightarrow \frac{dy}{dt} = -\frac{4}{3}\left(2\ \frac{\text{m}}{\text{sec}}\right)$

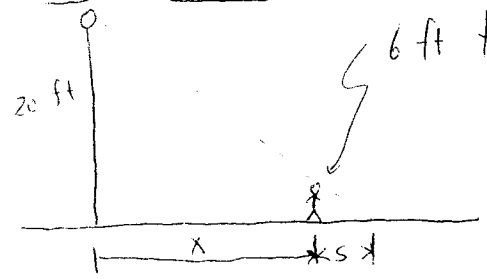
$\Rightarrow \boxed{\frac{dy}{dt} = -\frac{8}{3}\ \frac{\text{m}}{\text{sec}}}$

Example 3:

A 6 ft tall person is walking away from a streetlight that is 20 ft high, at the rate of 7 ft/sec. At what rate is the person's shadow increasing?

GENERAL SITUATION

1)



2) BY SIMILAR TRIANGLES

$$\frac{x+s}{20} = \frac{s}{6}$$

$$\Rightarrow 6x + 6s = 20s$$

$$\Rightarrow s = \frac{3}{7}x$$

3) $$\frac{ds}{dt} = \frac{3}{7}\frac{dx}{dt}$$

SPECIFIC SITUATION

4) $\frac{dx}{dt} = 7\ \frac{ft}{sec}$ $\quad \frac{ds}{dt} = ?$

5) $$\frac{ds}{dt} = \frac{3}{7}\left(7\ \frac{ft}{sec}\right)$$

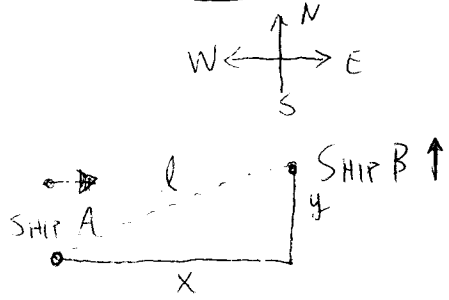
$$\Rightarrow \boxed{\frac{ds}{dt} = 3\ \frac{ft}{sec}}$$

Example 4:

At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4pm.

GENERAL SITUATION

1)



2) $$x^2 + y^2 = l^2$$

3) $$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2l\frac{dl}{dt}$$

SPECIFIC SITUATION

4) AT NOON, $x = 150$ Km $\quad y = 0$

AT 4pm, since distance = (rate)(time)

$(35\ \frac{km}{hr})(4\ hrs) = 140$ Km

$\Rightarrow$ AT 4pm, $x = 10$ Km

$\boxed{\frac{dx}{dt} = -35\ \frac{km}{hr}}$ $\quad \boxed{\frac{dy}{dt} = 25\ \frac{km}{hr}}$

AT 4pm, $d = rt \Rightarrow y = (25)(4) = 100$ Km

$\Rightarrow l^2 = x^2 + y^2 = (10)^2 + (100)^2 = 10{,}100\ km^2$

$\Rightarrow l = \sqrt{10{,}100}$ Km

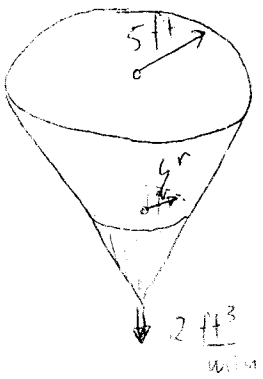
$\frac{dl}{dt} = ?$

5) $$\boxed{\frac{dl}{dt} = \frac{(10)(-35) + (100)(25)}{\sqrt{10{,}100}} = \frac{2150}{\sqrt{10{,}100}}\ \frac{km}{hr}}$$

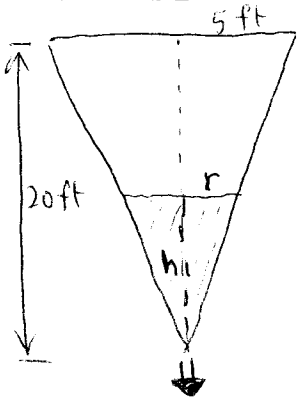
Example 5:
A tank filled with water is in the shape of an inverted cone 20 ft high with a circular base, on top, whose radius is 5 ft. Water is running out of the bottom of the tank at a constant rate of 2 ft³/min. How fast is the water level falling when the water is 8 ft deep?

GENERAL SITUATION

1)



SIDE VIEW



2) $V = \frac{1}{3}\pi r^2 h$

BY SIMILAR TRIANGLES

$\frac{r}{h} = \frac{5}{20} \Rightarrow r = \frac{1}{4}h$

So, $V = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h \Rightarrow V = \frac{\pi}{48}h^3$

3) $\frac{dV}{dt} = \frac{3\pi}{48}h^2 \cdot \frac{dh}{dt} = \frac{\pi h^2}{16}\frac{dh}{dt}$

SPECIFIC SITUATION

4) $\frac{dV}{dt} = -2\frac{ft^3}{min}$ $h = 8ft$

$\frac{dh}{dt} = ?$

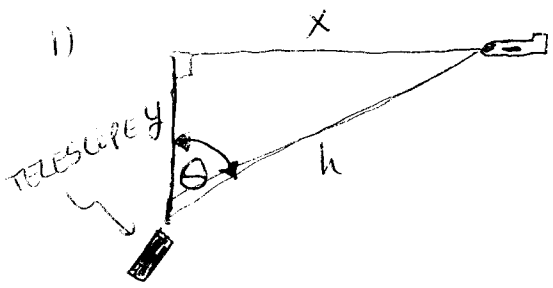
5) $\frac{dh}{dt} = \frac{16}{\pi h^2}\frac{dV}{dt} = \frac{16}{\pi 64}(-2)$

$\Rightarrow \boxed{\frac{dh}{dt} = -\frac{1}{2\pi}\frac{ft}{min}}$

Example 6:
A person is standing on the ground watching a jet through a telescope as it approaches at a speed of 10 miles/minute at an altitude of 7 miles. At what rate (in radians/minute) is the angle of the telescope changing when the horizontal distance of the jet from the woman is 24 miles? When the jet is directly above the person?

GENERAL SITUATION

1)



2) $\tan\theta = \frac{x}{y}$, NOTE THAT y IS CONSTANT

$\Rightarrow x = y\tan\theta$

3) $\frac{dx}{dt} = y\sec^2\theta\frac{d\theta}{dt}$

SPECIFIC SITUATION

4) ~~7~~ $y = 7$ miles $\frac{dx}{dt} = -10\frac{miles}{min}$

$x = 24$ miles $\frac{d\theta}{dt} = ?$

NOTE THAT $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{y}{h}} = \frac{h}{y}$

where $h = \sqrt{x^2 + y^2}$

5) $\frac{d\theta}{dt} = \frac{dx}{dt}\cos^2\theta\frac{1}{y}$

$= (-10)\left(\frac{y}{h}\right)^2\left(\frac{1}{y}\right) = \frac{-10y}{h^2}$

NOTE: USE $x = 24$ and $x = 0$ TO FIND THE NUMERICAL ANSWERS

$\boxed{\frac{d\theta}{dt} = \frac{(-10)(7)}{x^2+y^2} = \frac{-70}{x^2+49}\frac{radians}{min}}$