

INTRODUCTION TO SIGMA NOTATION

1. THE NOTATION ITSELF

Sigma notation is a way of writing a sum of many terms, in a concise form. A sum in sigma notation looks something like this:

$$\sum_{k=1}^5 3k$$

The Σ (sigma) indicates that a sum is being taken. The variable k is called the index of the sum. The numbers at the top and bottom of the Σ are called the upper and lower limits of the summation. In this case, the upper limit is 5, and the lower limit is 1. The notation means that we will take every integer value of k between 1 and 5 (so 1, 2, 3, 4, and 5) and plug them each into the summand formula (here that formula is $3k$). Then those are all added together.

$$\sum_{k=1}^5 3k = 3 \cdot 1 + 3 \cdot 2 + 3 \cdot 3 + 3 \cdot 4 + 3 \cdot 5 = 45$$

Example 1. Write out what is meant by the following:

$$\sum_{k=0}^3 \frac{1}{k+1}$$

Here, the index k takes the values 0, 1, 2, and 3. We'll plug those each into $\frac{1}{k+1}$ and add them together.

$$\sum_{k=0}^3 \frac{1}{k+1} = \frac{1}{0+1} + \frac{1}{1+1} + \frac{1}{2+1} + \frac{1}{3+1}$$

Example 2. Write out what is meant by the following:

$$\sum_{i=1}^8 (-1)^i$$

Date: Friday March 31, 2016.

The index variable here is written as i instead of k . That's ok. The most common variables to use for indexes include i , j , k , m , and n .

$$\begin{aligned}\sum_{i=1}^8 (-1)^i &= (-1)^1 + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8 \\ &= -1 + 1 + -1 + 1 + -1 + 1 + -1 + 1 \\ &= 0\end{aligned}$$

Try one on your own.

Example 3. Write out what is meant by the following (no need to simplify):

$$\sum_{n=-1}^4 \sqrt{n+1}$$

Let's try going the other way around.

Example 4. Write the following sum in sigma notation.

$$2 + 4 + 6 + 8 + \dots + 22 + 24$$

Notice that we can factor a 2 out of each term to rewrite this sum as

$$2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + 2 \cdot 4 + \dots + 2 \cdot 11 + 2 \cdot 12$$

That means that we are adding together 2 times every number between 1 and 12. The sigma notation could be

$$\sum_{k=1}^{12} 2k$$

There is no need to use k as our index variable. We could have just as easily used m or j instead.

$$\sum_{k=1}^{12} 2k = \sum_{m=1}^{12} 2m = \sum_{j=1}^{12} 2j$$

Notice, that these are NOT the same as

$$\sum_{k=1}^{12} 2m$$

Example 5. Write the following sum in sigma notation.

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots - \frac{1}{64} + \frac{1}{128}$$

This one is a little more complicated. We'll worry about the signs later, first we'll deal with the numbers themselves. Do you notice a pattern in the terms? Sure, we get to the next term by dividing by 2. That is:

$$\begin{aligned} 1 &= \frac{1}{2^0} = \left(\frac{1}{2}\right)^0 \\ \frac{1}{2} &= \frac{1}{2^1} = \left(\frac{1}{2}\right)^1 \\ \frac{1}{4} &= \frac{1}{2^2} = \left(\frac{1}{2}\right)^2 \\ &\dots \\ \frac{1}{128} &= \frac{1}{2^7} = \left(\frac{1}{2}\right)^7 \end{aligned}$$

If we call our index variable k , then k should go from 0 to 7, and the numbers themselves are just $\left(\frac{1}{2}\right)^k$. Now we need to deal with the signs. We say above that $(-1)^k$ will alternate between +1 and -1. That is, if we multiply our terms from above by $(-1)^k$, they will alternate between + and -. We are starting with $k = 0$, so $(-1)^0 = +1$ will give us the alternation starting at the sign we want.

$$\sum_{k=0}^7 (-1)^k \left(\frac{1}{2}\right)^k = \sum_{k=0}^7 \left(\frac{-1}{2}\right)^k$$

Try one on your own.

Example 6. Write the following sum in sigma notation.

$$\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{3}} + \frac{3}{\sqrt{4}} - \frac{4}{\sqrt{5}} + \dots + \frac{51}{\sqrt{52}} - \frac{52}{\sqrt{53}}$$